Generic attack on Duplex-Based AEAD Modes using Random Function Statistics

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Introduction

A generic attack against duplex-based AEAD modes

- A **forgery** attack
  in most cases, the key is recovered as well
- Based on **random function statistics**
  Previous works: average behaviour (see for example [BGW18])
  Our work: average and **exceptional** behaviour

Our contribution

- Improving knowledge of the **security of duplex-based modes**
- Breaking a security claim of XOODYAK [DHPVAVK20]
  (XOODYAK still meets the security requirement of NIST's LWC competition)

Duplex-based AEAD modes

Authenticated Encryption with Associated Data

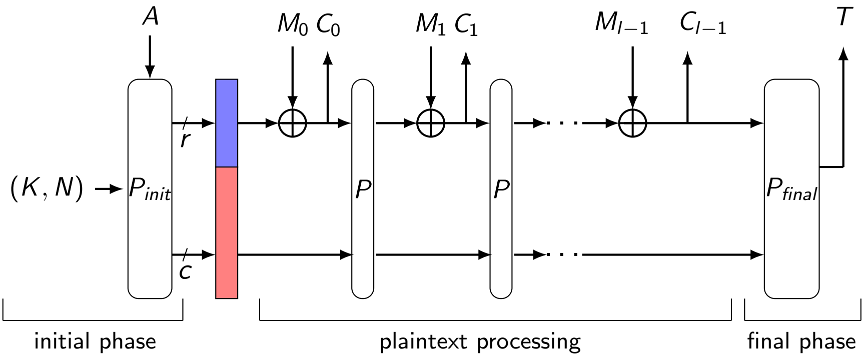
- Either **block-cipher based**: (tweakable) block cipher + mode
- Or **permutation-based**: public permutation + keyed mode
  Ex: XOODYAK = XOODOO[12] + Cyclist [DHPVAVK20]

Duplex-based modes of operation

- Permutation-based modes introduced by Bertoni, Daemen, Peeters, Van Assche [BDPVA11]
- An adaptation to the AEAD context of the **Sponge construction** [BDPVA07]

  Ex: SPONGEWRAP [BDPVA11], MonkeyWrap (KETJE) [BDPVAVK14], *etc.*

Duplex-based AEAD modes [BDPVA11]



- Permutation P operates on a state of length $b = r + c$ bits, where r is the **rate** and c the **capacity**
- First r bits : the **outer state**
- Next c bits : the **inner state**

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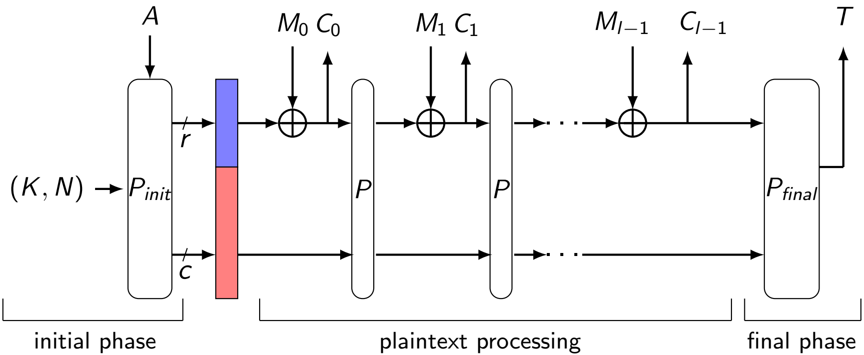

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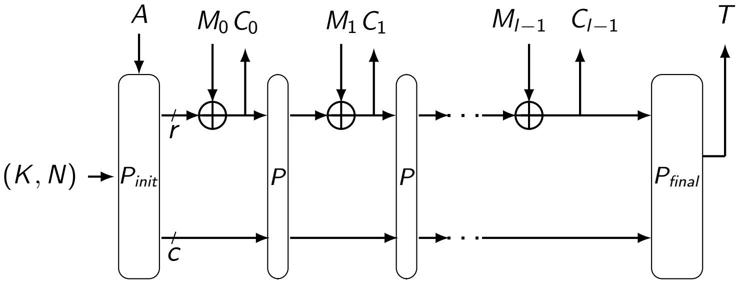
Ex: XOODYAK
r = 192
c = 192

Forgery attack on duplex-based modes

- **Privacy** and **integrity** are required in AEAD.
- It is assumed that: - the adversary is **nonce-respecting**
  - there is **no release of unverified plaintext**
- **Forgery attack:** find a decryption query (N, A, C, T) s.t. the tag verification succeeds (the decryption oracles returns the plaintext)

Encryption



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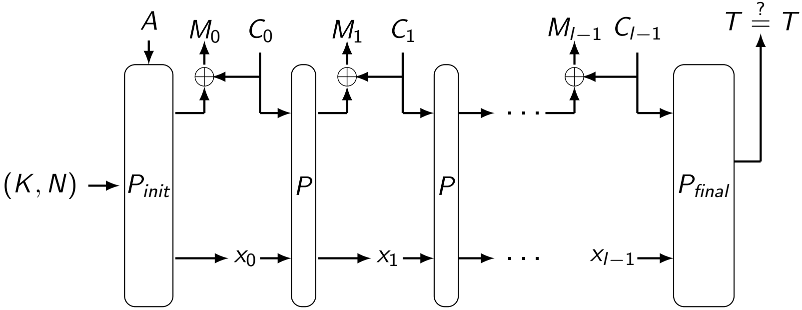

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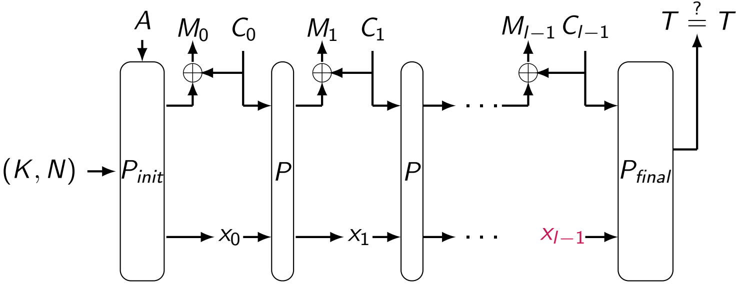

Guessing x_{l-1} allows to build a forgery!

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Total time complexity of an attack

$$\mathscr{T} = \sigma_e + \sigma_d + q_P + t_{extra-op}$$

where

σ_e is the number of online calls to P caused by encryption queries

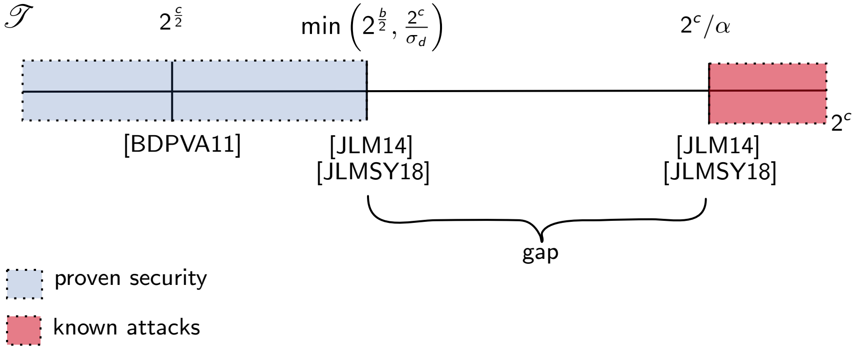
σ_d is the number of online calls to P caused by forgery attempts

q_P is the number of offline queries to P or P^{-1}

Our motivation

Disclaimer
this is (extremely) simplified

Assuming a sufficiently large key/tag length:



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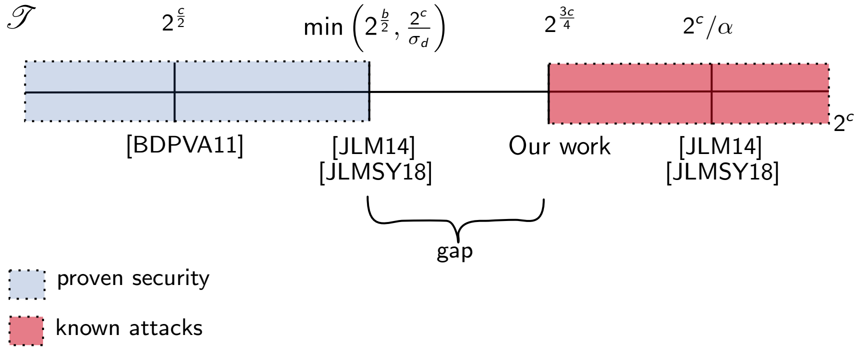

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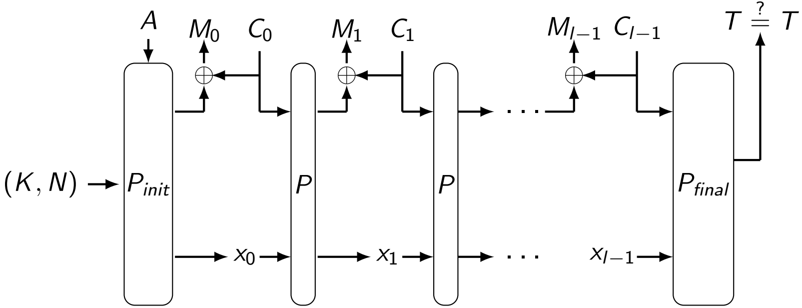
Main observation

Decrypting the ciphertext/tag pair $(C = C_0 \mid\mid \cdots \mid\mid C_{l-1}; T)$



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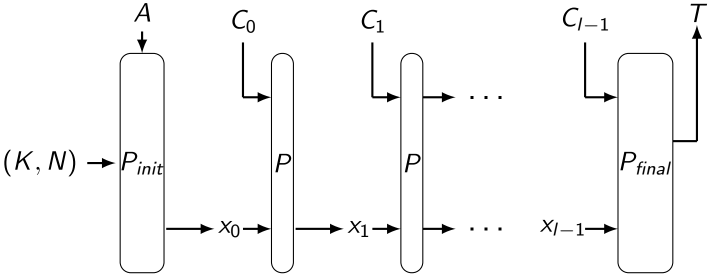

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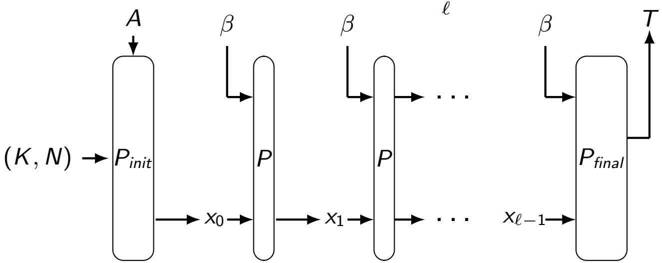

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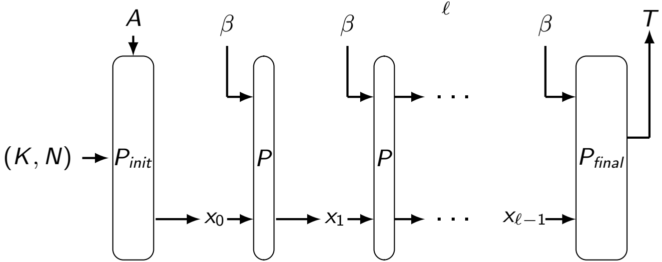
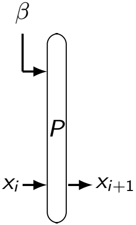


The tag verification iterates the function $F_\beta : \mathbb{F}_2^c \to \mathbb{F}_2^c$



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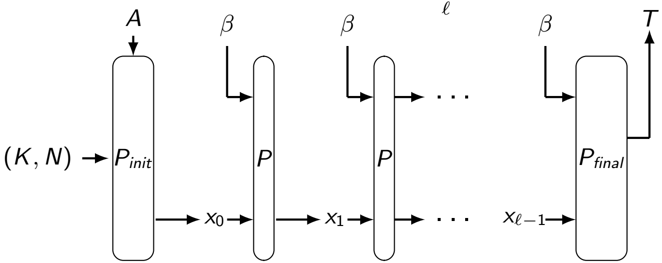

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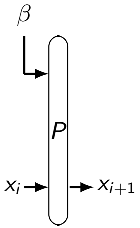

- For a random β, we expect F_β to behave as a **random function** drawn in $\mathfrak{F}_{2^c}$.
- For each nonce, we expect x_0 to behave as a **random point** drawn in the graph of F_β.

Graph of a random function F in $\mathfrak{F}_{2^c}$

Def: node i goes to node j iff $F(i) = j$.

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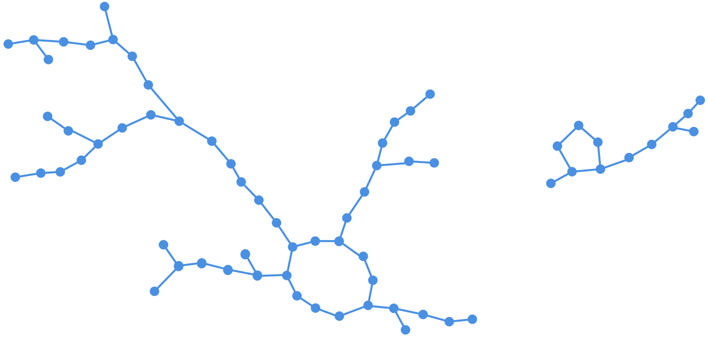

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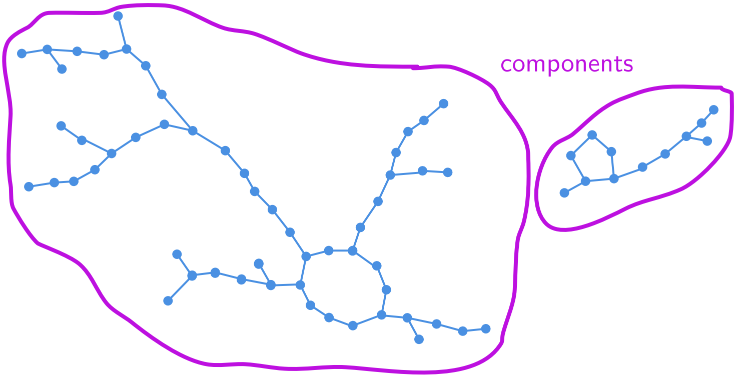

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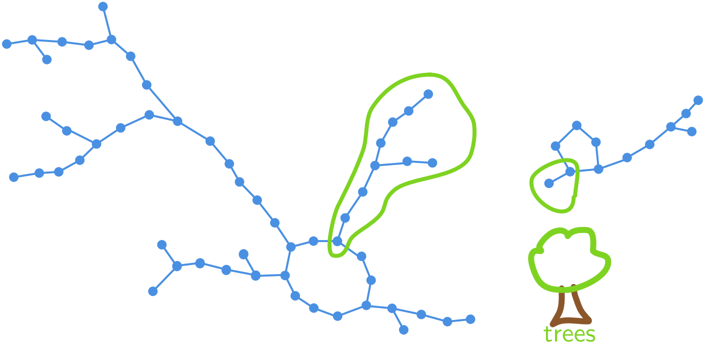

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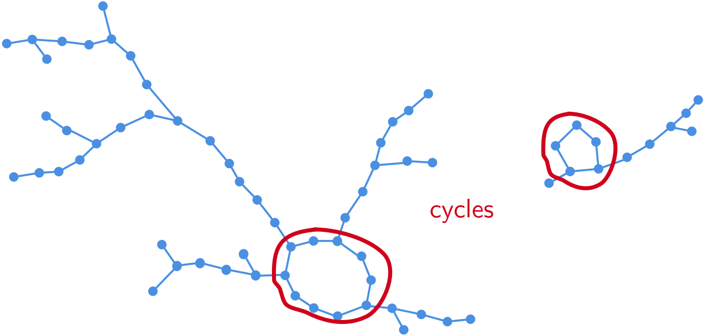

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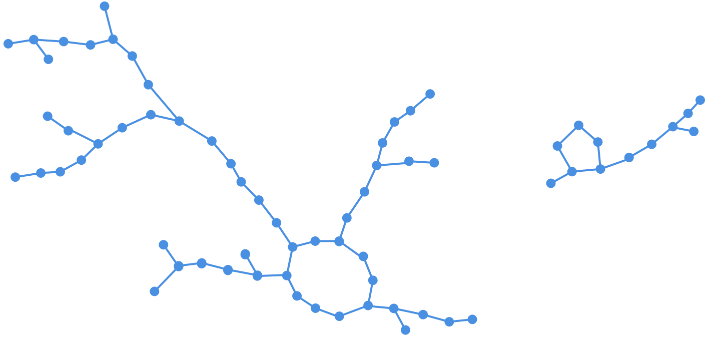

Average. . .

- Size of the largest component: $2^c \times 0.76$. [FO89]
- Cycle/tail length of a random point: $2^{\frac{c}{2}}\sqrt{\pi/8}$

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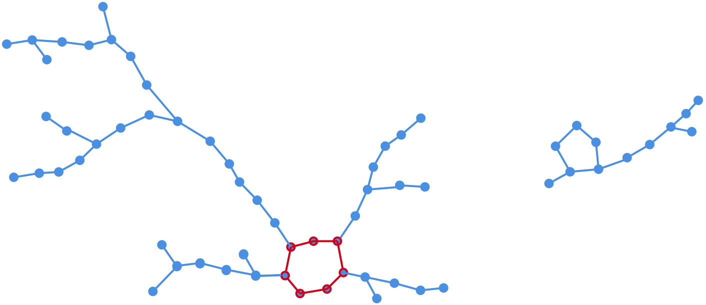


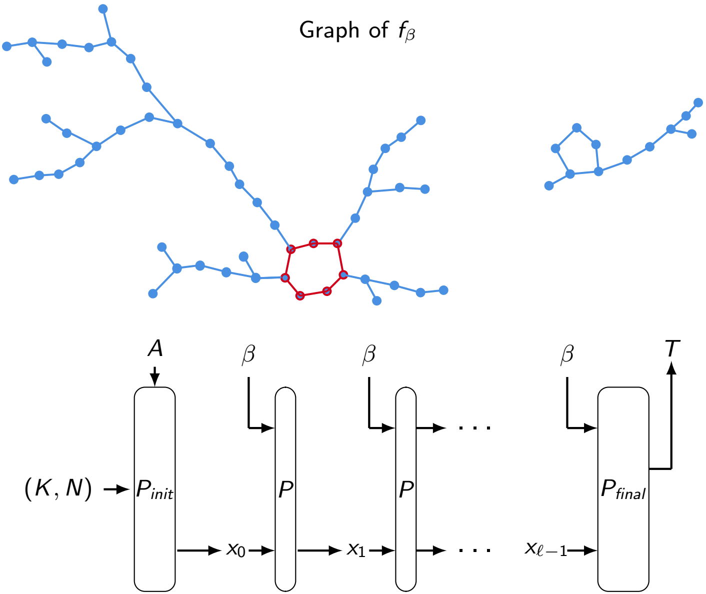
The probability that a random function has a component

- of cycle length at most $\leq 2^{\frac{c}{2}-\nu} \rightarrow$ its cycle is **exceptionally small**:
- of size at least $\geq 2^c \times s \rightarrow$ this component is **reasonably large**;

$$p_{s,\nu} \approx \sqrt{\frac{2(1-s)}{\pi s}} 2^{-\nu}$$ [DeLaurentis87]

Ex: proba for $s = 65\%$ and $\nu = \frac{c}{4}$ (cycle of length $\leq 2^{\frac{c}{4}}$): $0.6 \times 2^{-\frac{c}{4}}$

Core idea of our forgery attack



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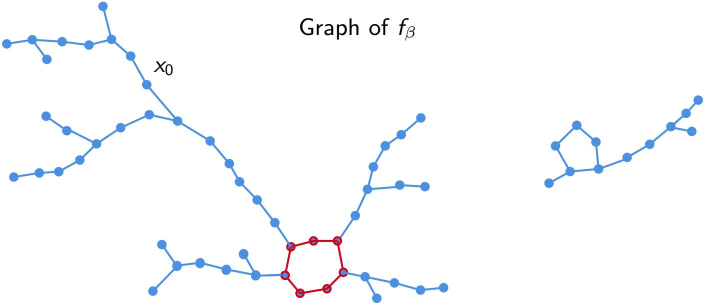

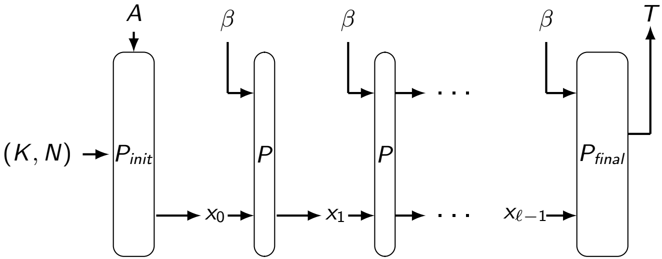

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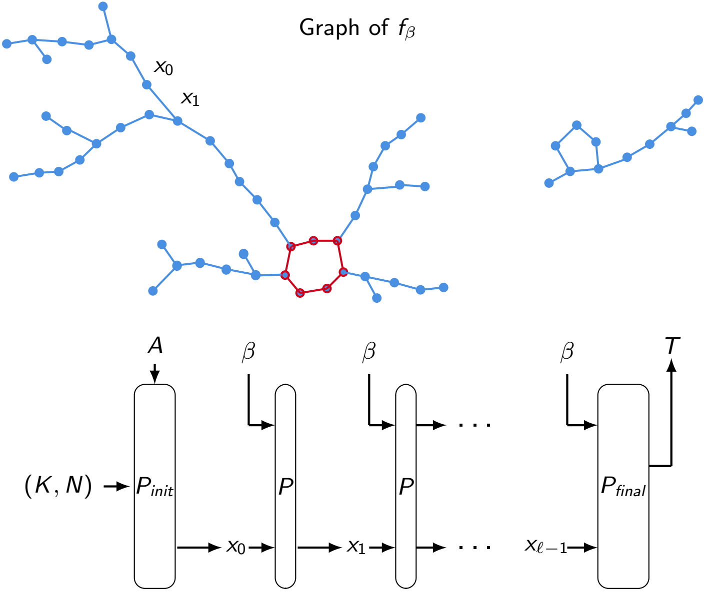

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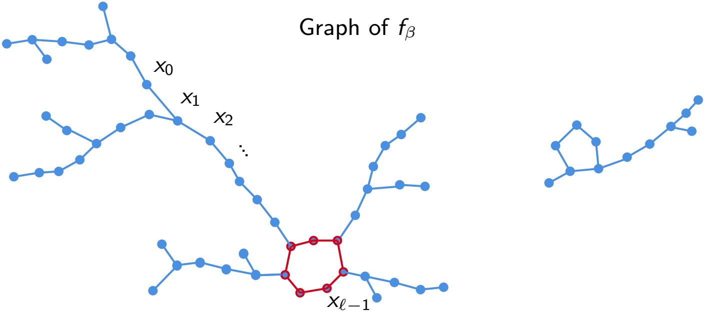

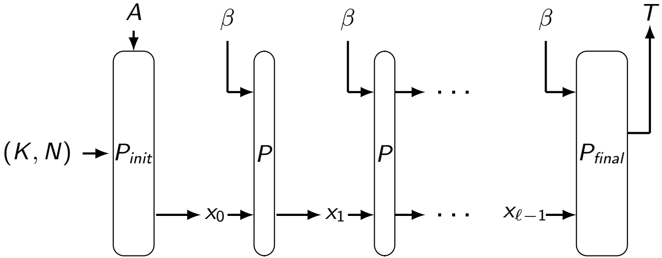

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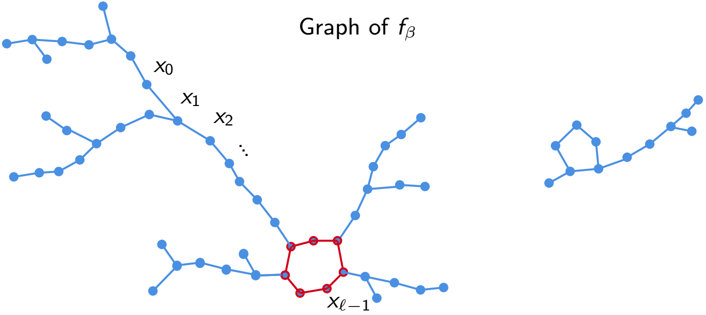

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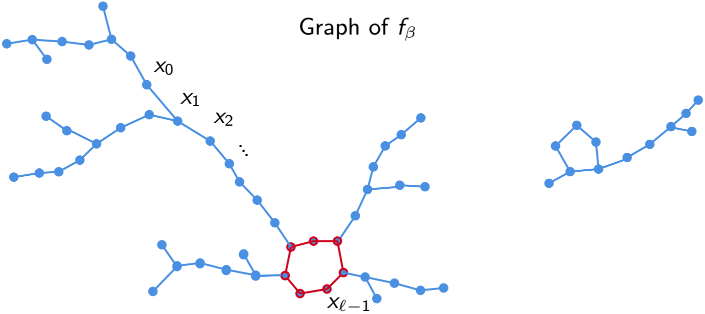

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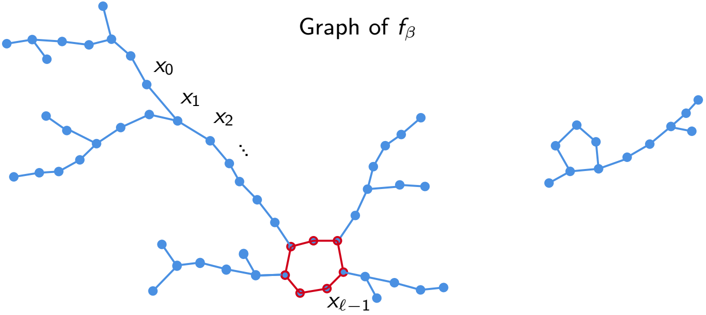

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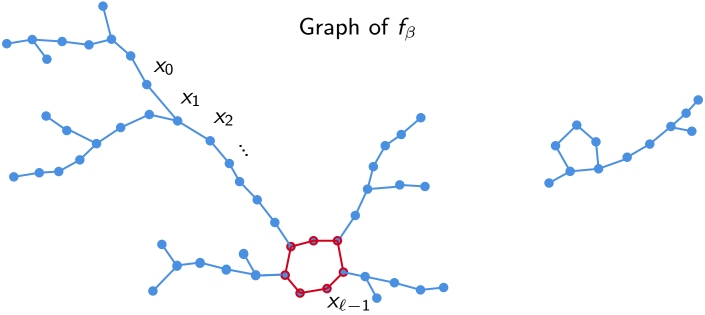

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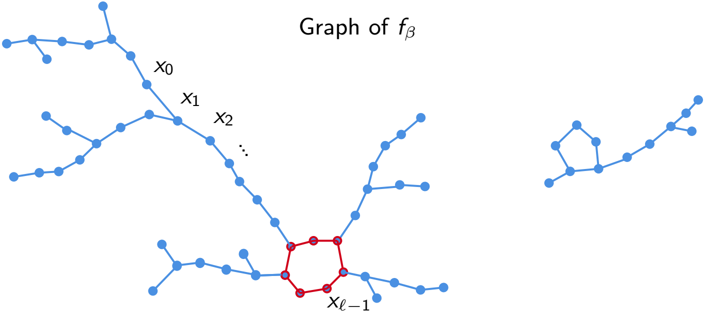

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Resulting forgery attack: try the $\leq 2^{\frac{c}{4}}$ possible values for T.

Core idea of our forgery attack

Precomputation phase
Find β s.t. F_β has a **large component** ($\geq 0.65 \times 2^c$) with an exceptionnally **small cycle** ($\leq 2^{\frac{c}{4}}$), recover this cycle

} **key independant**

Online phase
Submit $(N, A, C = \underbrace{\beta || \cdots || \beta}_{\ell}, T)$ queries to the decryption oracle where:

- N is randomly sampled
- A is set to the empty string
- ℓ is 'big enough' ($\approx 2^{\frac{c}{2}}$)
- $T = P_{final}(\beta || x)$, for x in the small cycle

Simplified complexity analysis (precomputation phase)

Precomputation phase: Find β s.t. F_β has a **large component** ($\geq 0.65 \times 2^c$) with an exceptionnally **small cycle** ($\leq 2^{\frac{c}{4}}$), recover this cycle

Complexity analysis:

- Drawing about $1/p_{s,\nu} \approx 2^{\frac{c}{4}}$ random β's
- For each β, investigating F_β costs $\approx 2^{\frac{c}{2}}$ per β thanks to Floyd's algorithm.

The total complexity is $\approx 2^{\frac{3c}{4}}$ **applications of** P.

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Note: the algorithm includes a test that the component is likely to be large enough.

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Note: At the cost of a more expensive prec. phase, the complexity of this step can be brought close(r) to $2^{\frac{c}{2}}$.

Small scale experiments

- Our attack is somewhat heuristic based.

→ Ex: corroborate that the F_β behave as **random functions** in practice.

- We implemented experiments with XOODOO[12] as P.

- All our practical results match our heuristic-based results.

→ Ex: the average tail length for a random F_β matches the average tail length for a random permutation.

- We also implemented the **precomputation algorithm**.

→ We found some **valid β values** for c up to 40.

Summary of our results

Our attack

- has **total time complexity** $\leq 21 \times 2^{\frac{3c}{4}}$;
- a **probability of success** $\geq 95\%$;
- can be transformed into a **key recovery** at a negligible extra cost if P_{init} is reversible (**how**: using the plaintext);
- is applicable to the modes of Norx v2, Ketje, KNOT and Keyak
- breaks the 184-bit security claim made by the designers of Xoodyak with an attack of complexity 2^{148}.

Preventing the attack

Two main features frustrate our cryptanalysis:

- **Key-dependent final phase.** (ASCON, NORX v3)

→ a correct guess on $x_{\ell-1}$ cannot be transformed into a forgery

- **No outer state overwriting.** (Beetle, SPARKLE, Subterranean)

→ the decryption of $\underbrace{\beta || \cdots || \beta}_{\ell}$ does not correspond to the iteration of a function

Thank you for your attention :)

Any questions?