The Key Recovery Step in Differential Attacks

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Context: symmetric cryptography

- 'Classical' cryptanalysis families: differential, linear, integral, ...
- New designs must come with arguments of resistance to each family.
- Difficulty to know which attack will be the most efficient.
  → Analysing a primitive is thus time-consuming, error-prone.
- In competitions: many ad-hoc cryptanalysis.
  → Difficult to outline generic criteria.

A direction: Proposing generic and automatic cryptanalytic tools.

Context: differential cryptanalysis

- Introduced by Biham and Shamir in 1990.
- One of the oldest and most famous cryptanalysis families

Yet, some primitives are still broken by differential cryptanalysis today.

- Some aspects of differential cryptanalysis are still not well-understood.
- The key recovery step is one of these aspects.

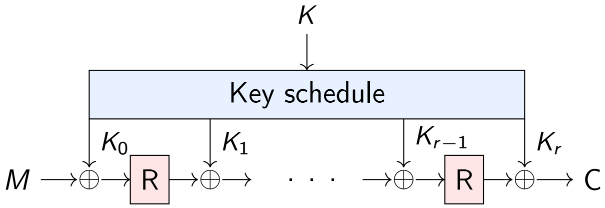
This talk/work: an attempt at providing some clarity.

This talk

- Key recovery attacks against block ciphers
- . . . using differential cryptanalysis
- . . . focusing on the key recovery step.

Key recovery attacks against block ciphers

General structure of an iterated block cipher



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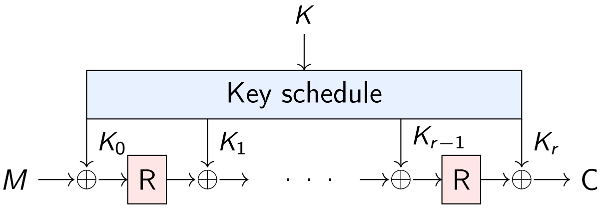
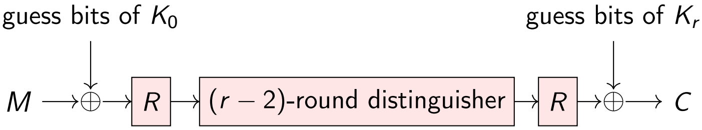


Key recovery attacks



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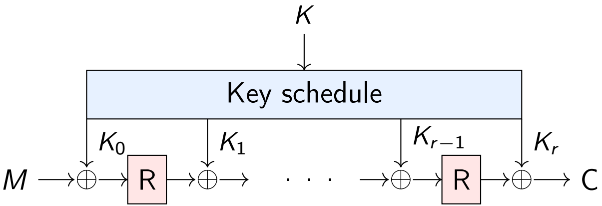

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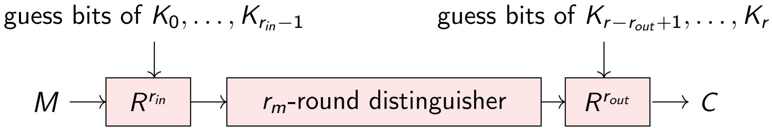


Outline

1. Differential Cryptanalysis of Block Ciphers
2. Our Model of the Core Key Recovery Step
3. A Generic Algorithm for the Core Key Recovery Step
4. Applications

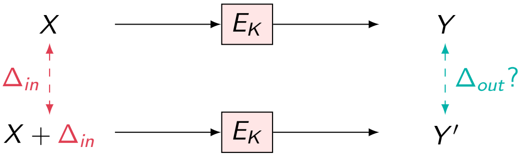
Differential cryptanalysis

For a block cipher E, a differential is a pair of input/output differences $(\Delta_{in}, \Delta_{out})$.

The probability of $(\Delta_{in}, \Delta_{out})$ is the probability p that

$$E_K(X) + E_K(X + \Delta_{in}) = \Delta_{out} \ ,$$

for a key K and an X both chosen uniformly at random.



If $p \gg 2^{-n}$, where n is the block size, then we have a differential distinguisher on E.

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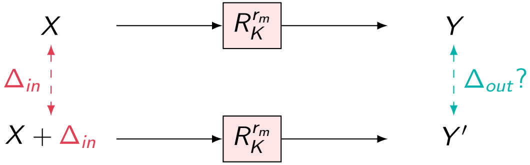

If $p \gg 2^{-n}$, where n is the block size, then we have a differential distinguisher on R^{r_m}.

Differential key recovery attacks

A differential distinguisher can be used to mount a key recovery attack.

- New primitives should come with arguments of resistance by design against this technique.
- Most of the arguments used rely on showing that differential distinguishers of high probability do not exist after a certain number of rounds.
- Not always enough: A deep understanding of how the key recovery works is necessary to claim resistance against these attacks.

The example of SPEEDY

SPEEDY-7-192 (Leander, Moss, Moradi, Rasoolzadeh, TCHES 21) is a 7-round block cipher.

Designers claim :

- 'The probability of any differential characteristic over **6 rounds** is $\leq 2^{-192}$.
- 'Not possible to add more than one key recovery round to any differential distinguisher.'

Better Steady than Speedy: Full Break of SPEEDY-7-192. Boura, David, Heim Boissier, Naya-Plasencia. **EUROCRYPT 2023**

- Distinguisher over 5.5 rounds ($\rightarrow$ of proba 0 [BN24]).
- Key recovery on 1.5 rounds.
- This work motivated us to work more specifically on the key recovery step.

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In previous works

The key recovery step is often done

- either in a 'naive' and non-efficient way;
- or using a tedious and error-prone procedure.

Emergence of new tools for cryptanalysis.

- most tools focus on the search for a differential distinguisher;
- the key recovery step is often considered using heuristics (e.g. [DF16]).

Our contribution: KYRYDI

A Generic Algorithm for Efficient Key Recovery in Differential Attacks - and its Associated Tool.
Boura, David, Derbez, Heim Boissier, Naya-Plasencia. **EUROCRYPT 2024**

Automatic key recovery for SPN block ciphers with

- a bit-permutation as linear layer;
- an (almost) linear key schedule.

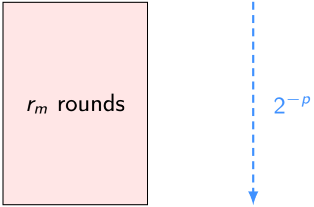
Link to our tool **KYRYDI**:

`https://gitlab.inria.fr/capsule/kyrydi`

Differential key recovery attacks

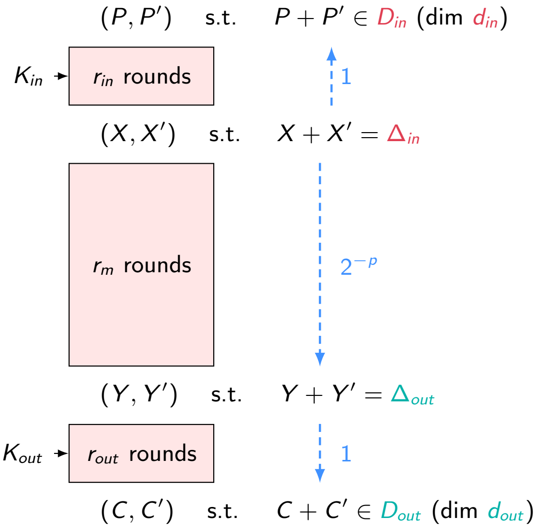
Differential distinguisher

(X, X') s.t. $X + X' = \Delta_{in}$



(Y, Y') s.t. $Y + Y' = \Delta_{out}$

Differential key recovery attacks



Differential key recovery attacks

(P, P') s.t. $P + P' \in D_{in}$ (dim d_{in})

$K_{in} \rightarrow$ r_{in} rounds — 1

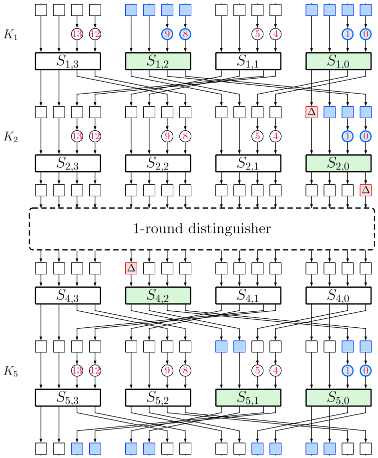
(X, X') s.t. $X + X' = \Delta_{in}$

r_m rounds — 2^{-p}

(Y, Y') s.t. $Y + Y' = \Delta_{out}$

$K_{out} \rightarrow$ r_{out} rounds — 1

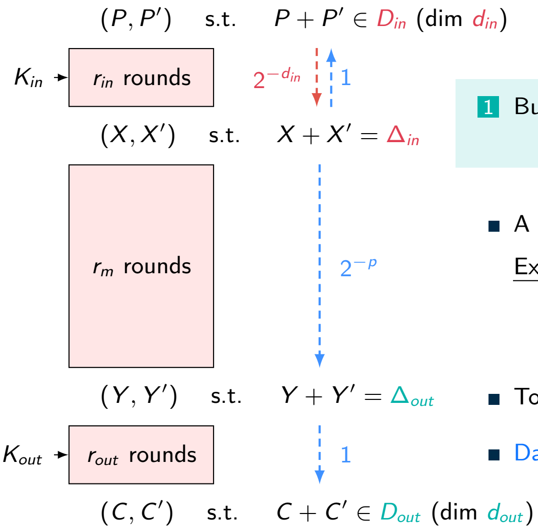
(C, C') s.t. $C + C' \in D_{out}$ (dim d_{out})



Ex: $D_{in} = \{0\}^4 \times \mathbb{F}_2^4 \times \{0\}^4 \times \mathbb{F}_2^4$, $d_{in} = 8$.

$D_{out} = \{0\}^8 \times \mathbb{F}_2^8$ $d_{out} = 8$.

Differential key recovery attacks (1/3)



1 Build enough pairs for at least one to satisfy the differential.

i.e. $2^{p+d_{in}}$ pairs $((P, C), (P', C'))$ s.t. $P + P' \in D_{in}$.

- A structure of size $2^{d_{in}}$ allows to build $2^{2d_{in}}$ pairs.

  Ex: $D_{in} = \{0\}^4 \times \mathbb{F}_2^4 \times \{0\}^4 \times \mathbb{F}_2^4$, $d_{in} = 8$.

  - Structures of the form $\{c_1\} \times \mathbb{F}_2^4 \times \{c_2\} \times \mathbb{F}_2^4$ where $c_1, c_2 \in \mathbb{F}_2^4$.

- To build enough pairs, one needs $2^{p-d_{in}}$ such structures.

- Data complexity: 2^p plaintexts/ciphertext pairs.

Differential key recovery attacks (2/3)

(P, P') s.t. $P + P' \in D_{in}$ (dim d_{in})

K_{in} → r_{in} rounds $2^{-d_{in}}$ ↓ ↑ 1

(X, X') s.t. $X + X' = \Delta_{in}$

r_m rounds 2^{-p}

(Y, Y') s.t. $Y + Y' = \Delta_{out}$

K_{out} → r_{out} rounds 1

(C, C') s.t. $C + C' \in D_{out}$ (dim d_{out})

2 Filter out pairs that cannot follow the differential.

i.e. only retain the fraction $2^{d_{out}-n}$ of pairs s.t. $C + C' \in D_{out}$.

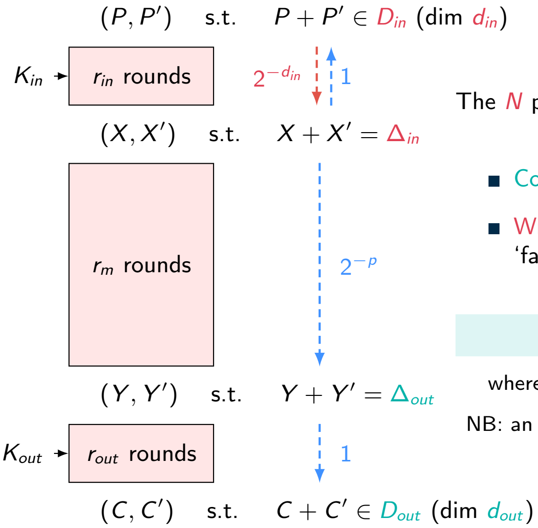
Ex: $D_{out} = \{0\}^8 \times \mathbb{F}_2^8$, $d_{out} = 8 \rightarrow$ filter 2^{-8}.

- Can be done e.g. using hash tables.
- Done for a cost at most 2^p i.e. the data complexity.

Number of pairs to consider in the key recovery step:

$$N = 2^{p+d_{in}+d_{out}-n}.$$

Differential key recovery attacks



The N pairs provide a test for each guess on the involved external key material:

- Correct key guess: one pair satisfies the differential.
- Wrong key guess: on average, $N \cdot 2^{-d_{in}-d_{out}} = 2^{p-n} \ll 1$ 'false alarm(s)'.

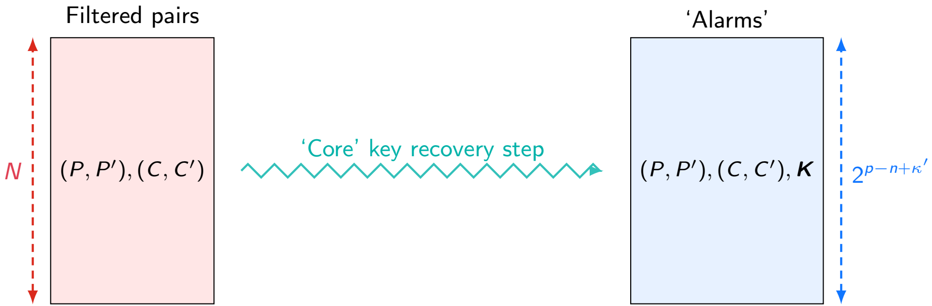
Remaining candidates: $2^{p-n+\kappa'} \ll 2^{\kappa'}$.

where κ' is the number of bits involved in the external key material.

NB: an exhaustive search on the remaining unknown key bits is required.

3. Core key recovery step

Procedure that allows to enumerate the alarms $((P, P'), (C, C'), \boldsymbol{K})$ as efficiently as possible.



$P + P' \in D_{in}$
$C + C' \in D_{out}$

$\boldsymbol{K} \in \mathbb{F}_2^{\kappa'}$ partially encrypts/decrypts
(P, P') to Δ_{in}, (C, C') to Δ_{out}

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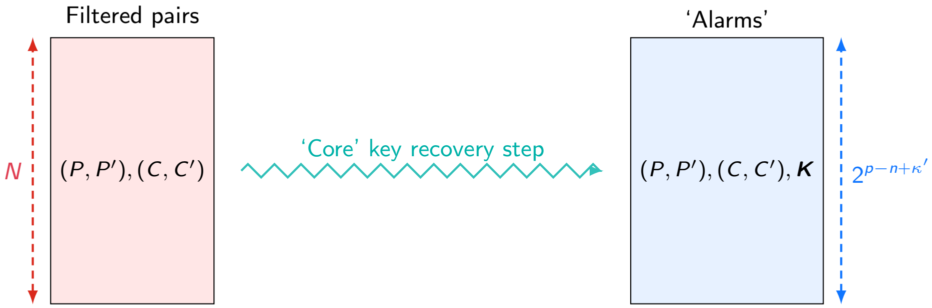

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What is the complexity of this procedure?

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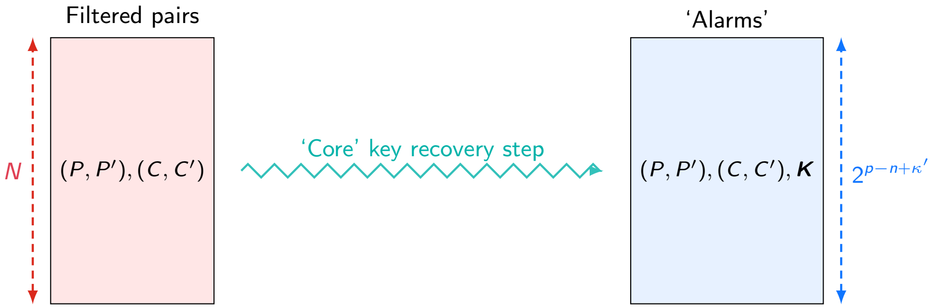

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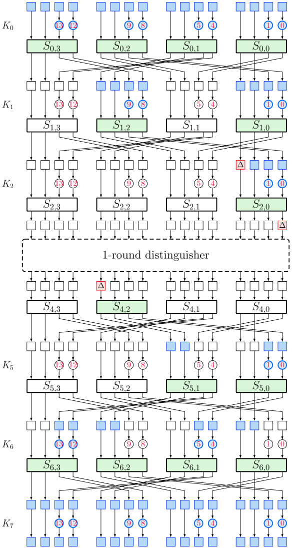
$\boldsymbol{K} \in \mathbb{F}_2^{\kappa'}$ partially encrypts/decrypts (P, P') to Δ_{in}, (C, C') to Δ_{out}

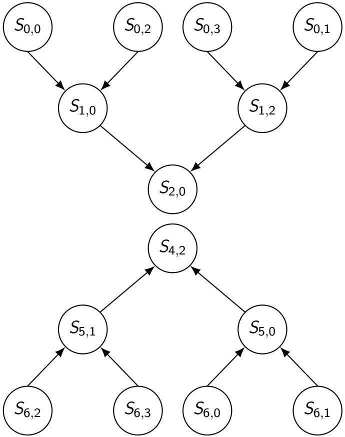
What is the complexity of this procedure?

- Upper bound: $\min(2^{\kappa}, N \cdot 2^{\kappa'})$
- Lower bound: $N + 2^{p-n+\kappa'}$

Outline

The key recovery problem as a graph

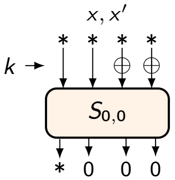




'Solving' an active S-box: For a given pair, finding the guesses on the key material that allow it to respect the differential constraints.

'Solving' S-boxes : the example of $S_{0,0}$

A solution to S is any tuple (x, x', k) s.t. $x + x' \in \nu_{in}$ and $S(x + k) + S(x' + k) \in \nu_{out}$.

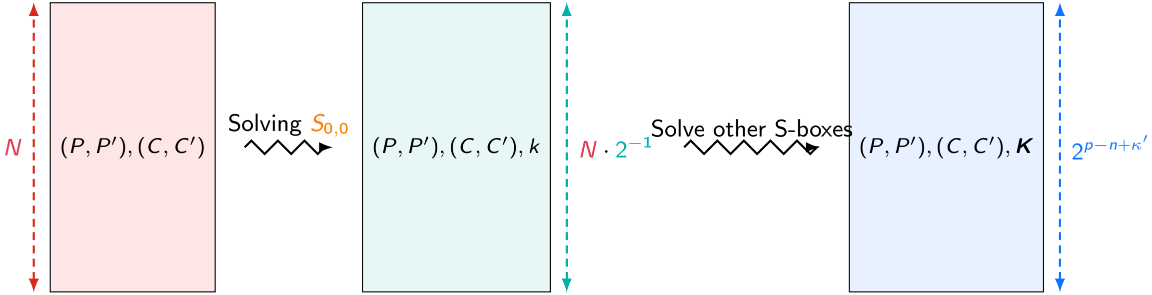


- Number of solutions (x, x', k) to $S_{0,0}$: $2^{4+1+2} = 2^7$.
- $S_{0,0}$ is an S-box of the first round :
  On any of the N pairs, the plaintext pair determines the value of (x, x').
- Probability to match a solution is $c_i = 2^7 \cdot 2^{-8} = 2^{-1}$.

Solving $S_{0,0}$ filters $N \cdot 2^{-1}$ triplets with a determined value on 2 key bits.

Goal: Reduce the number of triplets as early as possible whilst maximizing the number of determined key bits.

'Solving' S-boxes



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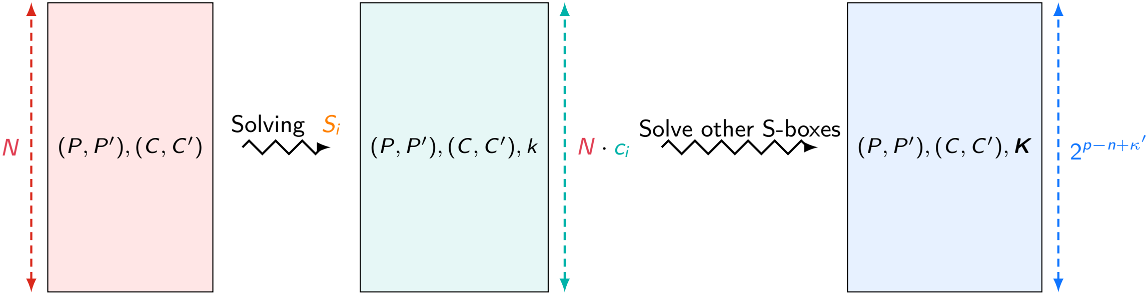

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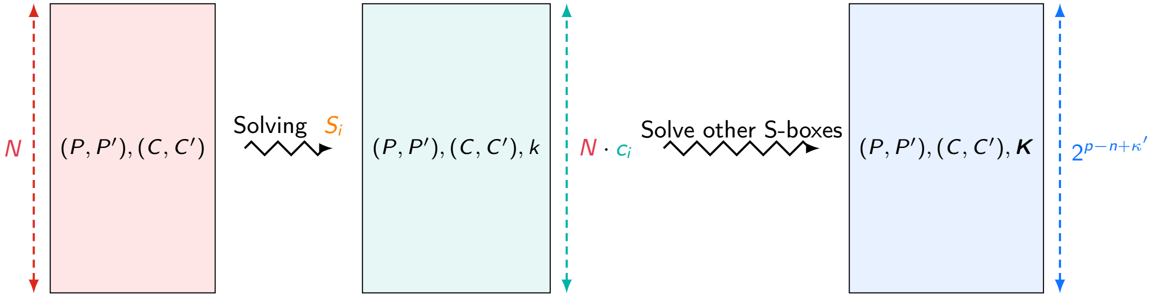
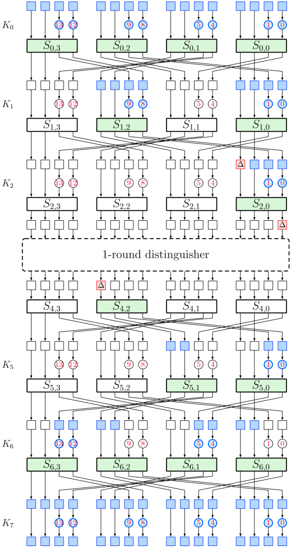
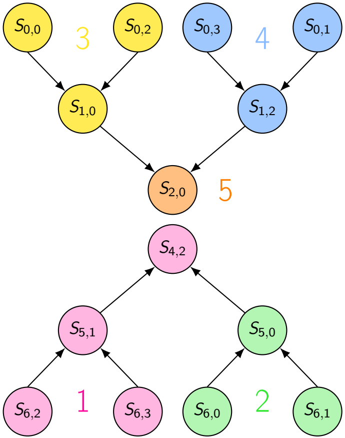


This can be generalised to any subset of active S-boxes!

The key recovery problem as a graph





Key recovery = partition of the nodes + associated order

Outline

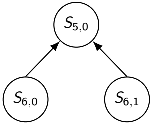
Considering strategies

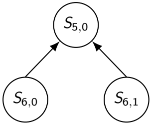
Strategy $\mathscr{S}_X$ for a subgraph X

Procedure that defines a partition of X and an order in which each subgraph in the partition is solved.

Parameters of a strategy $\mathscr{S}_X$:

- number of solutions s_X
- online time complexity $T(\mathscr{S}_X)$





A strategy can be further refined with extra information: e.g. memory, offline time.

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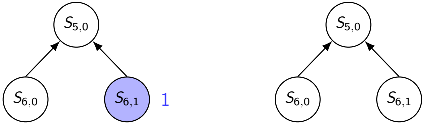

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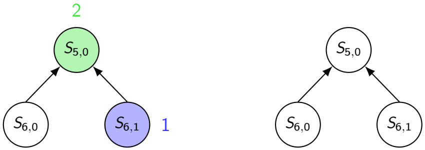

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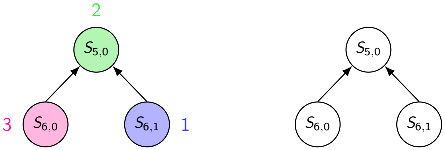

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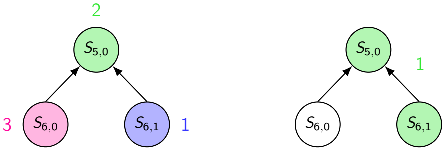

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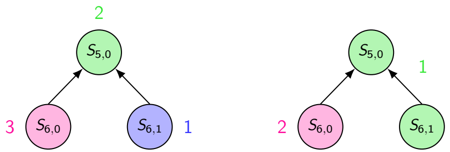

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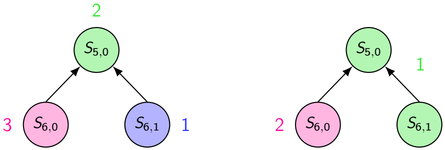

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Goal: Build an efficient strategy for the whole graph.

- Based on basic strategies: strategies for a single S-box and an 'initial N pairs' strategy $\mathscr{O}$.

Merging two strategies

Assuming that $s_X < s_Y$, the merge $\mathscr{S}'$ of $\mathscr{S}_X$ and $\mathscr{S}_Y$ is the strategy which consists in

1. running $\mathscr{S}_X$, store the solutions in a hash table;
2. running $\mathscr{S}_Y$, and for each solution, look for matches.

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Parameters of $\mathscr{S}'$

- $s_{X \cup Y} = s_X + s_Y - \#$ bit-relations between the nodes of X and Y ⚠ log scale
- $T(\mathscr{S}') \approx \max(T(\mathscr{S}_X), T(\mathscr{S}_Y), s_{X \cup Y})$

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An optimal strategy for a graph is obtained by merging two optimal strategies for two of its subgraphs.

A dynamic programming approach

'An optimal strategy for a graph is obtained by merging two optimal strategies for two of its subgraphs'

Dynamic programming approach:

- 'Clever' exhaustive search.
- Bottom-up approach: merge strategies with a small time complexity first.
- Keep only the optimal strategy found for each subgraph X.
- Restricting merges thanks to heuristics.

Comparing two strategies

Compare two strategies $\mathscr{S}_X^1$ and $\mathscr{S}_X^2$ for the same subgraph X

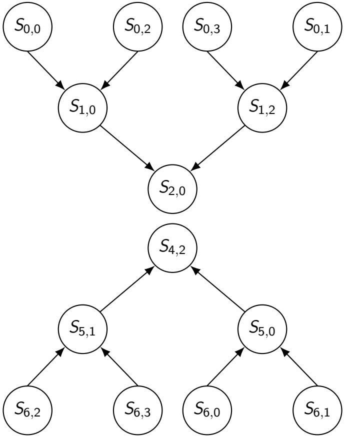
1. Choose the one with the best time complexity.
2. If same time complexity, choose the one with the best memory complexity.

Compare $\mathscr{S}_X^1$ and $\mathscr{S}_Y^2$ when $Y \subset X$

If the number of solutions and time complexity of $\mathscr{S}_X^1$ are not higher than those of $\mathscr{S}_Y^2$, then we can freely replace $\mathscr{S}_Y^2$ by $\mathscr{S}_X^1$.

Restricting merges (1/2)

1. Only allow merges between co-dependent sub-graphs:
   - An edge between two nodes;
   - Or at least a common node between two subgraphs.



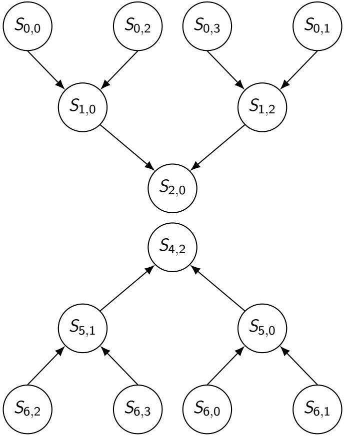
Examples:

- $\mathscr{S} = \{\mathscr{O}, S_{0,0}\}$ cannot be merged with $\mathscr{S}' = \{S_{1,2}\}$.
- $\mathscr{S} = \{\mathscr{O}, S_{6,0}\}$ can be merged with $\mathscr{S}' = \{S_{0,0}\}$.

Restricting merges (2/2)

2 A non-filtering node can be merged with an online strategy iff

- It is computed by partially encrypting/decrypting the data;
- Or it does not increase the number of solutions.



Examples:

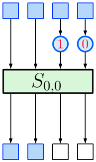
- $\mathscr{S} = \{\mathscr{O}, S_{0,0}, S_{0,2}\}$ can always be merged with $\mathscr{S}' = \{S_{1,0}\}$.
- $\mathscr{S} = \{\mathscr{O}, S_{0,0}\}$ can only be merged with $\mathscr{S}' = \{S_{1,0}\}$ if it does not increase the number of solutions.

Additional improvements (1/2) : Sieving

Idea: Use the differential constraints to filter out pairs that cannot follow the differential, regardless of the value of the key.

- Example: $(x_3, x'_3, x_2, x'_2, x_1 \oplus x'_1, x_0 \oplus x'_0)$

  Filter: $36/2^6 = 2^{-0.83}$.



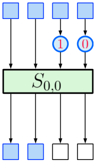
Additional improvements (1/2) : Sieving

Idea: Use the differential constraints to filter out pairs that cannot follow the differential, regardless of the value of the key.

- Example:

$$(\mathtt{x}_3, \mathtt{x}_3', \mathtt{x}_2, \mathtt{x}_2', \mathtt{x}_1 \oplus \mathtt{x}_1', \mathtt{x}_0 \oplus \mathtt{x}_0')$$

Filter: $36/2^6 = 2^{-0.83}$.



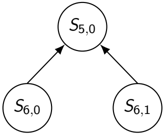
Pre-sieving

Apply a sieve on all S-boxes of the external rounds.

Advantage : The key recovery step is performed on $N' \leq N$ pairs.

Additional improvements (2/2) : Precomputing partial solutions

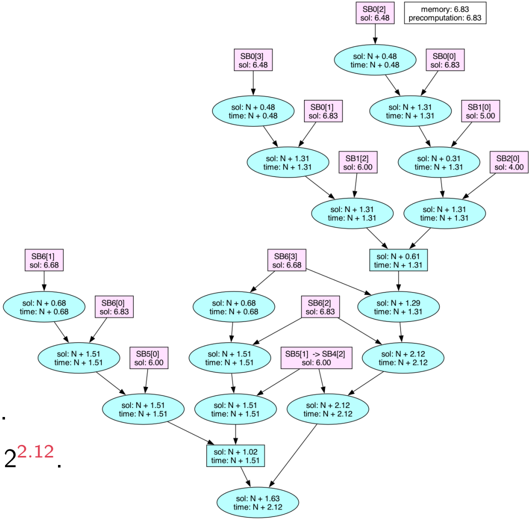
Idea: Precompute the partial solutions to some subgraph.



- Impact on the memory complexity and the offline time of the attack.
- The key recovery strategy found by the tool depends on how much memory and offline time are allowed.

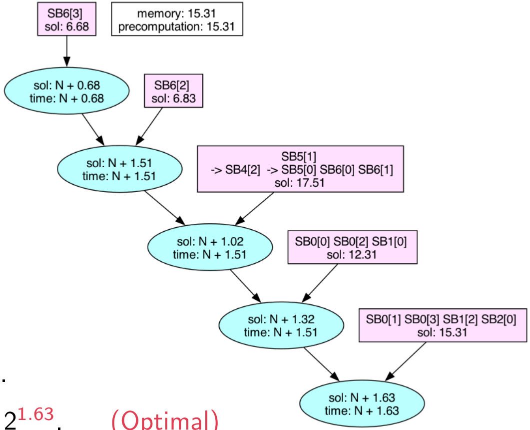
Outline

Application to the toy cipher



- **Nr solutions:** $N \cdot 2^{1.63}$.
- **Time complexity:** $N \cdot 2^{2.12}$.
- **Memory:** $N \cdot 2^{6}$.

Application to the toy cipher



- **Nr solutions:** $N \cdot 2^{1.63}$.
- **Time complexity:** $N \cdot 2^{1.63}$. (Optimal)
- **Memory:** $N \cdot 2^{15}$.

Applications

Start from an existing distinguisher that led to the best key recovery attack against the target cipher.

- `RECTANGLE-128`: Extended by one round the previous best attack.
  - From 18 to 19 rounds out of 25.
- `PRESENT-80`: Extended by two rounds the previous best differential attack.
  - From 16 to 18 rounds out of 31.
- `GIFT-64`: Best key recovery strategy without additional techniques.
  - 26 rounds out of 28.

Future improvements, open questions

- Taking into account key-schedule relations more accurately (including non-linear ones?).
- Incorporate tree-based key recovery techniques [Bro+21].
- Handle ciphers with more complex linear layers.
- Prove optimality.
- Generalise to other attacks.

The best distinguisher does not always lead to the best key recovery!

Ultimate goal

Combine the tool with a distinguisher-search algorithm to find the best possible attacks.

A dynamic programming approach

Simplified algorithm: Initialisation

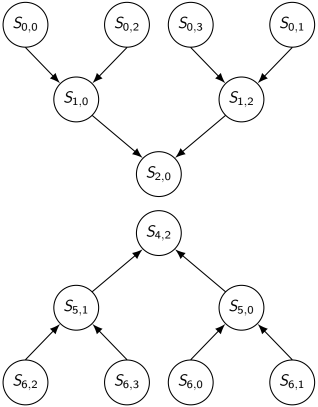
Create two lists:

- $\mathrm{L}_{done} \leftarrow \mathcal{O}$ where $\mathcal{O}$ corresponds to the 'initial N pairs' node.
- $\mathrm{L}_{current} \leftarrow$ basic strategies.

Ex: toy cipher:

$S_{0,0}, S_{0,1}, S_{0,2}, S_{0,3}, S_{1,0}, S_{1,2}, S_{2,0}, S_{6,0}, S_{6,1}, S_{6,2}, S_{6,3}, S_{5,0}, S_{5,1}, S_{4,2}$

NB: The 'online node' $\mathcal{O}$ is linked to all the plaintext/ciphertext nodes.



A dynamic programming approach

Simplified algorithm (2)

While $\mathrm{L}_{current} \neq \emptyset$:

- Let S be the strategy from $\mathrm{L}_{current}$ with the smallest T.
  - For any S' in L_{done} allowed to be merged with S:
    Let S'' be their merge.
    If no strategy from L_{done} nor $\mathrm{L}_{current}$ is better than S'':
    - Add S'' to $\mathrm{L}_{current}$.
    - Remove from both L_{done} and $\mathrm{L}_{current}$ all strategies worst than S''.
- Remove S from $\mathrm{L}_{current}$, add it to L_{done}.