Generic attacks using random functions statistics

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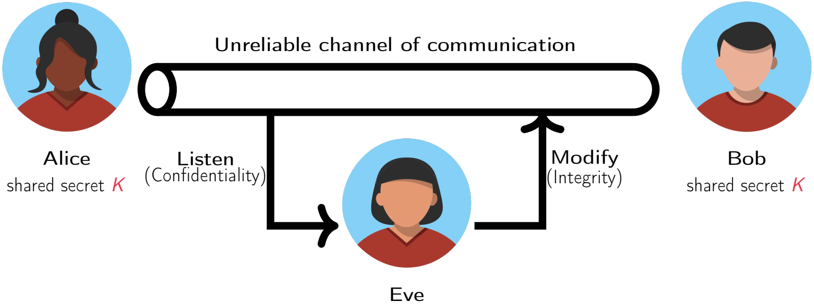
Symmetric cryptology

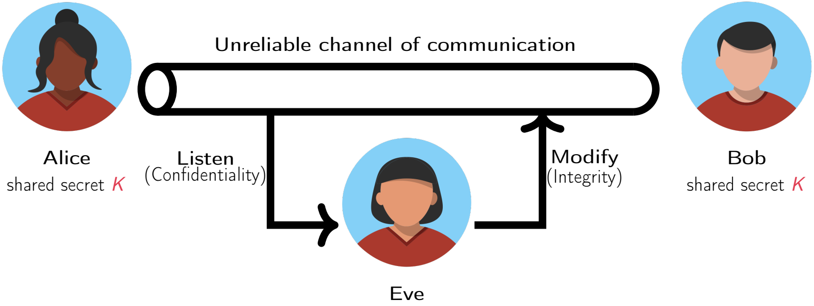
Symmetric cryptology studies algorithms allowing two entities that share a common secret, the key K, to communicate in a secure manner*



Symmetric cryptology

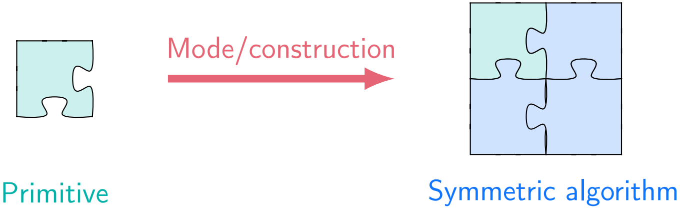
Symmetric cryptology studies algorithms allowing two entities that share a common secret, the key K, to communicate in a secure manner*

*... as well as some 'keyless' algorithms such as hash functions.



Building symmetric algorithms

Cryptography relies on building blocks called *primitives* used within *modes of operation* or *constructions* to build more complex algorithms.



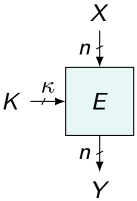
- The notion of primitive is *relative*.
- Most primitives do not provide a standalone cryptographic mechanism on their own.

Primitives

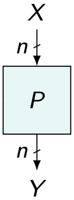
- A block cipher of key size κ bits and block size n bits is a function

$$\begin{array}{rccc} E & : & \mathbb{F}_2^\kappa \times \mathbb{F}_2^n & \longrightarrow & \mathbb{F}_2^n \\ & & (K, X) & \longmapsto & E(K, X) \end{array}$$

  such that for any key K, $E_K(\cdot) := E(K, \cdot)$ is a permutation of $\mathbb{F}_2^n$.

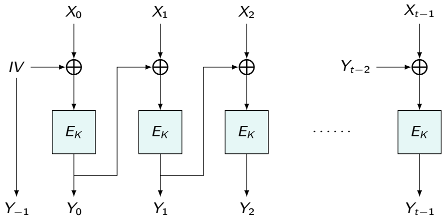


- A public permutation P over $\mathbb{F}_2^n$ does not depend on a key.



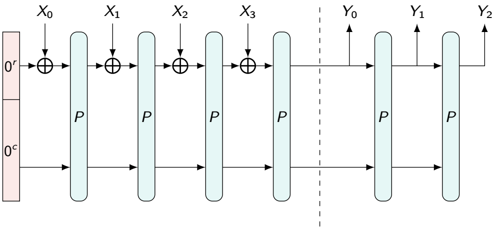
- **Block cipher-based**

  Ex: the encryption mode CBC.



- **Permutation-based**

  Ex: the sponge construction for hashing.



Security in cryptography (1/2)

Two main approaches:

- Provable security: reducing the security of a scheme to some ‘reasonable’ assumption.
  - How do we assess the reasonability of our assumption?
- Cryptanalysis: security analysis effort.
  - If the international cryptographic community cannot break it, then, hopefully, noone else can.
  - International standardisation competitions organised by the NIST.
  - The cryptanalysis effort should be global, continuous and comprehensive.

Security in cryptography (2/2)

Primitive security

- can only be guaranteed through cryptanalysis.

Mode/construction security

- Proved under the assumption that the primitive is secure.
- Proofs provide a partial information on the security level.
- Cryptanalysis, and in particular generic attacks, provides a complementary point of view.

A generic attack assumes an ideal behaviour of the underlying primitive.

Elementary ex: generic key recovery attack on E given X and $Y = E_K(X)$.

- Exhaustively try the 2^κ possible secret keys.

This talk

- Symmetric cryptanalysis.
- Generic attacks against a variety of iterated constructions:
  - Hash functions;
  - Message Authentication Codes (MAC) modes;
  - Authenticated encryption (AE) modes.
- Our main tool: random functions graphs statistics.

Outline

Random functions

$\mathfrak{F}_N$ is the set of functions which map a finite set of size $N \in \mathbb{N}^*$ to itself.

Our main focus:

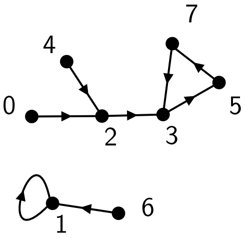
The graph of f, denoted by $G(f)$, is a directed graph such that an edge goes from node i to node j if and only if $f(i) = j$.

Properties and statistics of functional graphs are used in generic attacks.

Functional graphs: an example

The graph of f, denoted by $G(f)$, is a directed graph such that an edge goes from node i to node j if and only if $f(i) = j$.

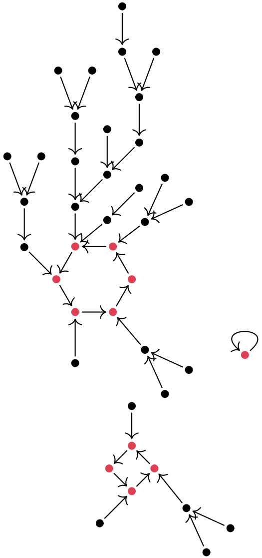
$$f : [\![0;7]\!] \longrightarrow [\![0;7]\!]$$
$$\begin{cases} 0 & \longmapsto 2 \\ 1 & \longmapsto 1 \\ 2 & \longmapsto 3 \\ 3 & \longmapsto 5 \\ 4 & \longmapsto 2 \\ 5 & \longmapsto 7 \\ 6 & \longmapsto 1 \\ 7 & \longmapsto 3 \end{cases}$$



Functional graphs (1)

Definitions.

- The graph of f can be seen as a set of connected components.
- Each connected component has a unique cycle.
- Each cyclic node is the root of a tree.



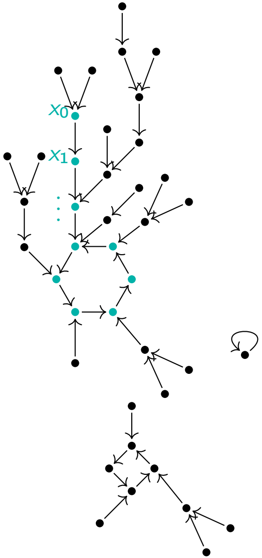
Statistics (e.g. [FO89]).

- Expected size of f's largest component: $0.76N$
- Expected size of f's largest tree: $0.48N$

Functional graphs (2)

For any $x_0 \in G(f)$

- $(x_i := f^i(x_0))_{i \in \mathbb{N}}$ is eventually periodic.
- $(x_i)_{i \in \mathbb{N}}$ graphically corresponds to a path linked to a cycle.



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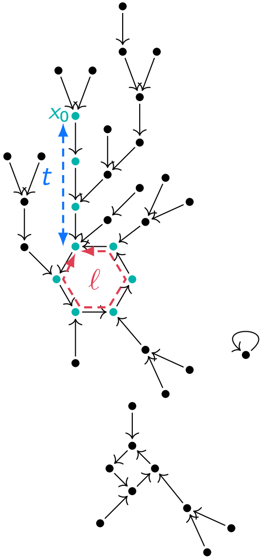
Definitions.

- Tail length $t(x_0)$: smallest i s.t. x_i is in the cycle.
- Cycle length $\ell(x_0)$: number of nodes in the cycle.

Statistics. For x a random node:

- Expected value of its tail length $t(x)$: $\sqrt{\pi N/8}$.
- Expected value of its cycle length $\ell(x)$: $\sqrt{\pi N/8}$.



Outline

Cryptographic hash functions

Definition. A cryptographic hash function is a function $H : \mathbb{F}_2^* \to \mathbb{F}_2^n$ such that

- Preimage resistance. Given $D \in \mathbb{F}_2^n$, it is difficult to find $M \in \mathbb{F}_2^*$ s.t. $H(M) = D$;
- Second preimage resistance. Given M, it is difficult to find $M' \neq M$ s.t. $H(M') = H(M)$;
- Collision resistance. It is difficult to find (M, M'), $M \neq M'$ such that $H(M) = H(M')$.

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Generic collision attack: Compute $H(M)$ for $O(2^{n/2})$ messages M, store M at the address $H(M)$.

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Generic collision attack: Compute $H(M)$ for $O(2^{n/2})$ messages M, store M at the address $H(M)$.

Memory complexity is also a $O(2^{n/2})$.

Solution: a generic memory-negligible collision attack using functional graphs.

A memory-negligible collision attack on H

Let $f \in \mathfrak{F}_{2^n}$ be defined as

$$f : \begin{array}{ccc} \mathbb{F}_2^n & \longrightarrow & \mathbb{F}_2^n \\ x & \longmapsto & H(x)\,. \end{array}$$

Step 1. A cycle finding algorithm allows to recover a cyclic node x_c

- in time $O(2^{n/2})$;
- using a negligible amount of memory.

Step 2. Using x_c, one can

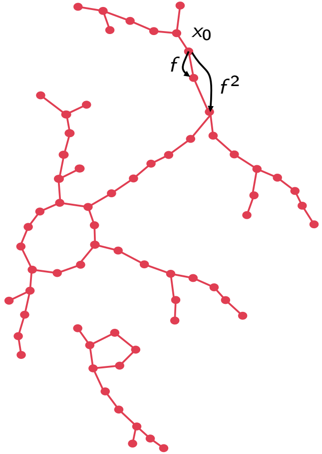
- recover the cycle length $\ell(x_c)$,
- find a collision on f, and thus on H,

in time $O(2^{n/2})$ and with negligible memory.

Ex: Floyd's cycle finding algorithm

parameters: $f \in \mathfrak{F}_{2^n}$

$x_0 \leftarrow_R \mathbb{F}_2^n$
$\mathit{turtle}, \mathit{hare} \leftarrow x_0, x_0$
for $i = 1$ to $2^n - 1$ **do**
    $\mathit{turtle} \leftarrow f(\mathit{turtle})$
    $\mathit{hare} \leftarrow f^2(\mathit{hare})$
    if $\mathit{turtle} = \mathit{hare}$ **then**
        return turtle
    end if
end for



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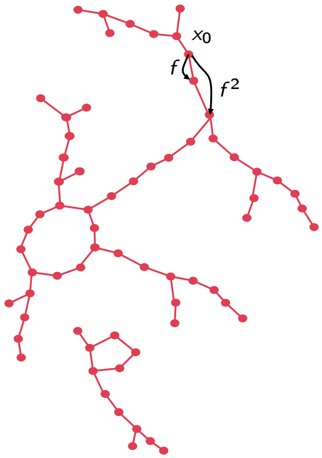
The tail length $t = t(x_0)$ is the smallest j s.t. x_j in the cycle.



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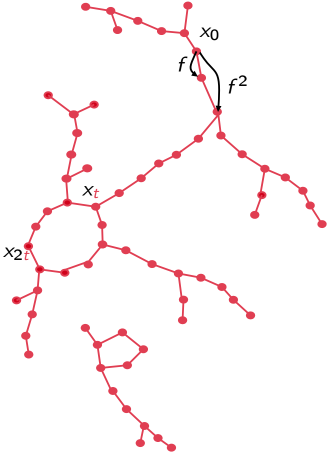

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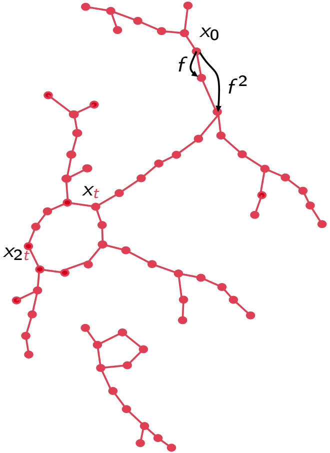
Let $d_t = dist(x_t, x_{2t})$.



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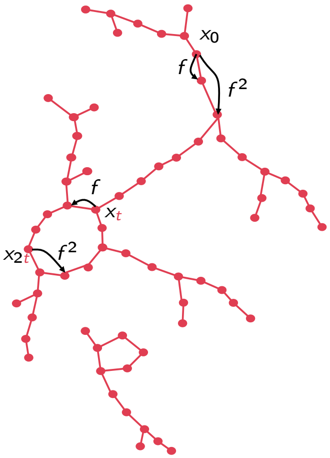
$$dist(f(x_t), f^2(x_{2t})) = d_t + 1 \mod \ell\,.$$



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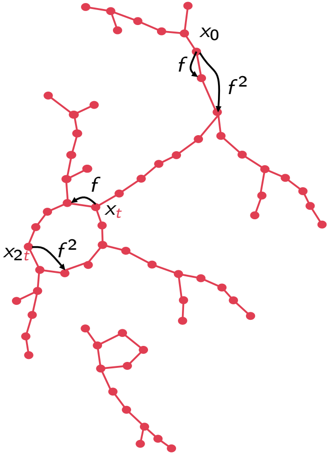
$$dist(x_{t+k}, x_{2(t+k)}) = d_t + k \mod \ell\,.$$



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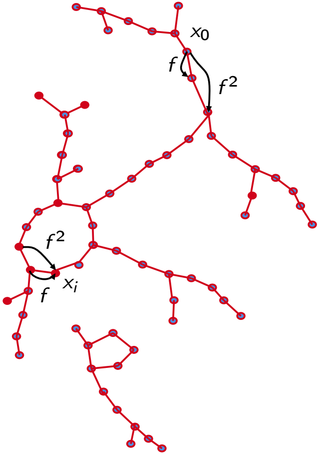
After at most $t + \ell$ tries, the algorithm finds i s.t. $x_i = x_{2i}$, and x_i is in the cycle.



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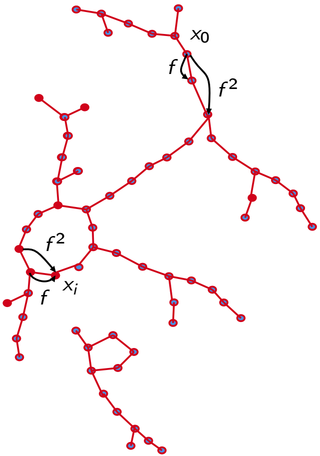
If f behaves like a random function, if x_0 is drawn at random, we expect $t = \ell = \sqrt{\pi/8} \cdot 2^{n/2}$.



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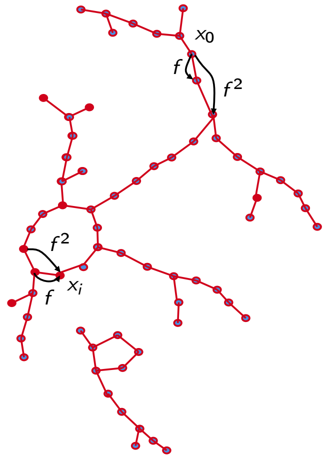
Floyd's time complexity: $O(2^{n/2})$ evaluations of f, memory complexity is negligible.



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Outline

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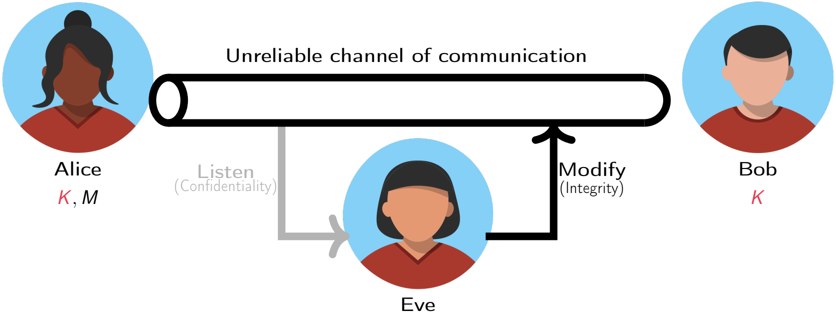
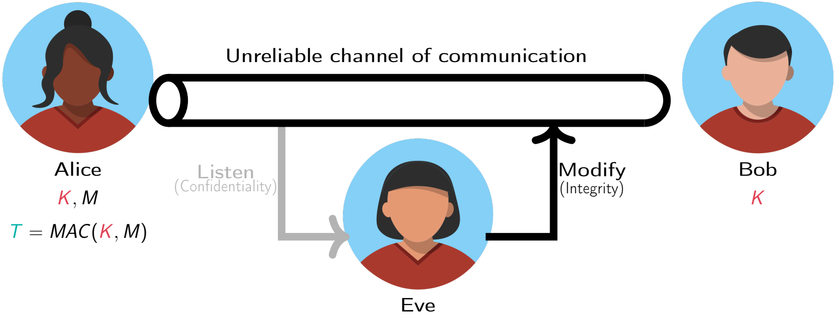


A Message Authentication Code algorithm (MAC) produces a fixed length tag that guarantees the integrity of the message.

Message Authentication Code (MAC) algorithms



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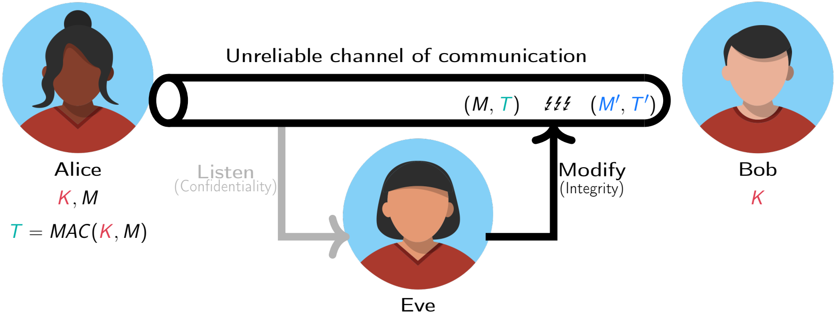

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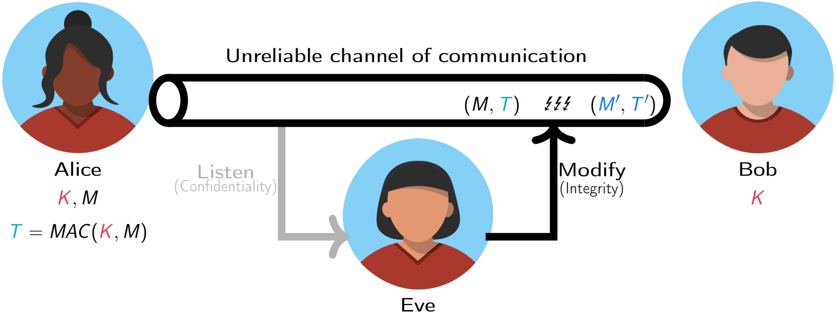

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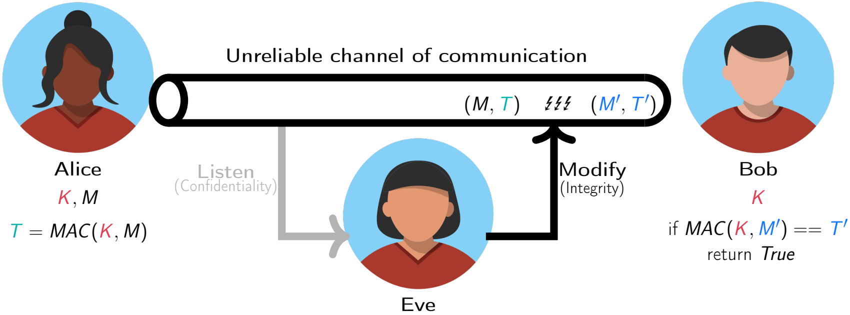


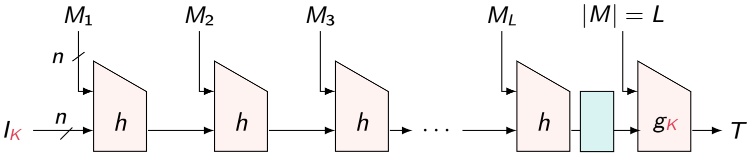
A Message Authentication Code algorithm (MAC) produces a fixed length tag that guarantees the integrity of the message.

Hash-based MACs

Hash functions can be used to build MACs.

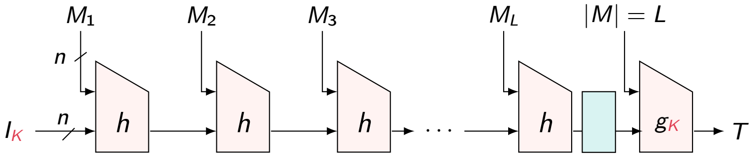
- It is easy to build a secure MAC with an ideal hash function, i.e. a random oracle.
- With a real hash function, it is essential to study generic attacks.
- Several papers analyse the generic security of HMACs.

We present a 2013 state recovery attack by Leurent, Peyrin and Wang on `HMAC` [BCK96].



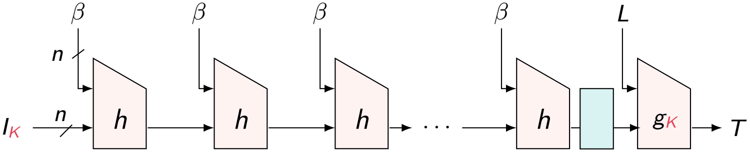
State-recovery attack on HMAC [LPW13]

$M = M_1 \,||\cdots||\, M_L$



State-recovery attack on HMAC [LPW13]

$$M = \beta \,||\cdots||\beta = \beta^L$$



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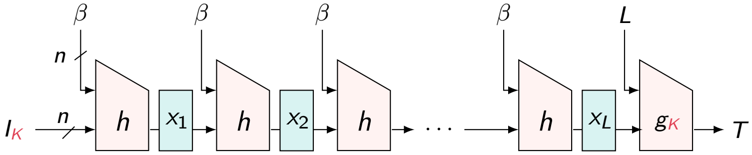

The tag generation iterates the function

$$\begin{aligned} h_\beta : \mathbb{F}_2^n &\longrightarrow \mathbb{F}_2^n \\ x &\longmapsto h(\beta, x)\,. \end{aligned}$$

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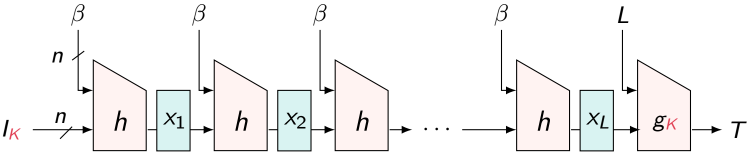
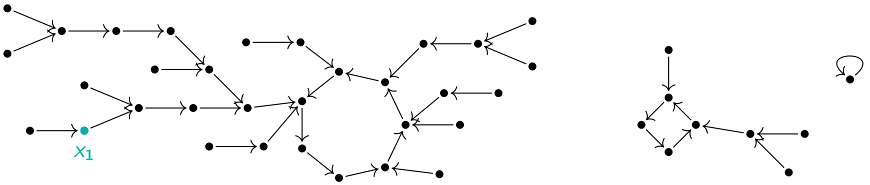


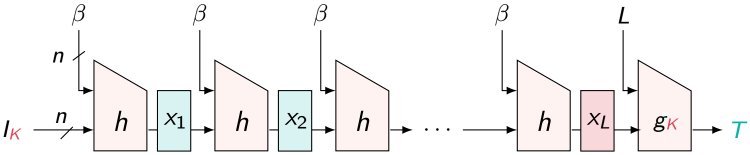
- For a random β, we expect h_β to behave as a function drawn at random in $\mathfrak{F}_{2^n}$.
  - Giant component with about 76% of the nodes.
- We expect x_1 to behave as a node drawn at random in the graph of h_β.
  - With proba 0.76, x_1 is in the giant component.
  - $t(x_1) = \ell(x_1) = \sqrt{\pi/8} \cdot 2^{n/2}$.

State-recovery attack on HMAC [LPW13]

- Setting $L = cst \cdot 2^{n/2}$:



Graph of h_β



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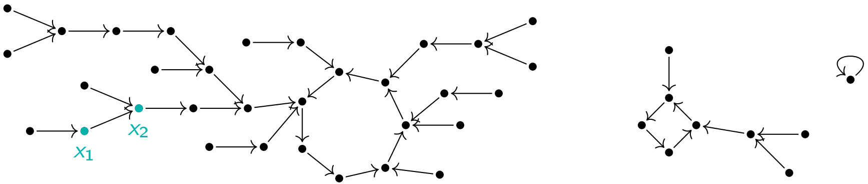

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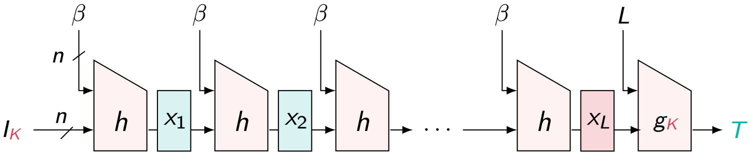

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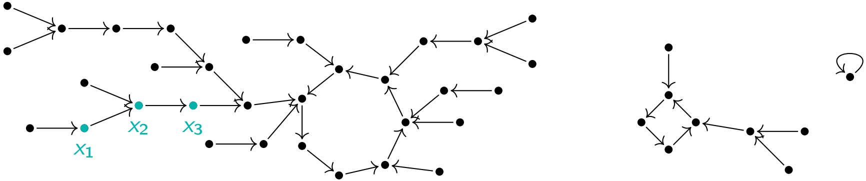

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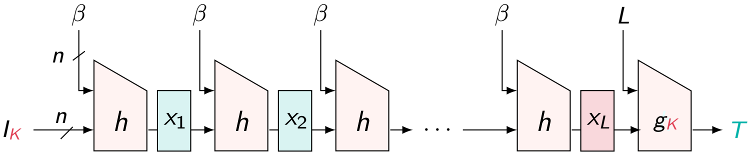

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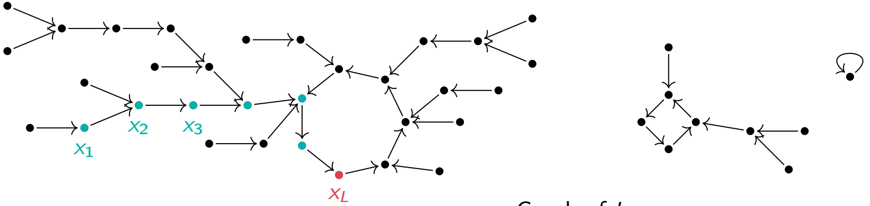
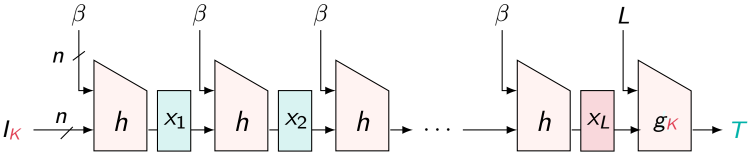
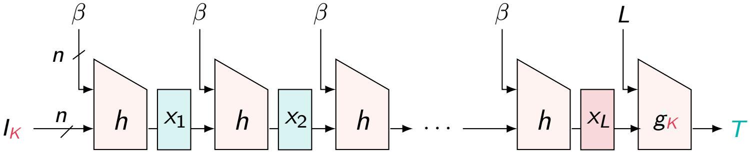
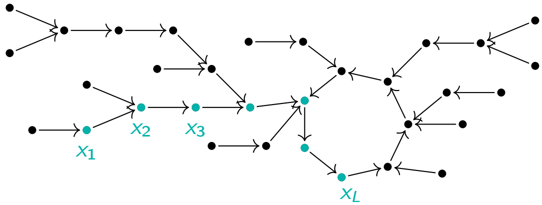


Graph of h_β



Idea 1: Building two messages who reach the same state





$M_1 = \beta^L$ and $M_2 = \beta^{L+\ell}$ reach the same final state.

Two issues: $\neq$ message lengths + the state is not recovered.

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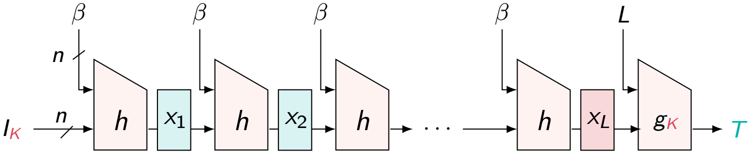
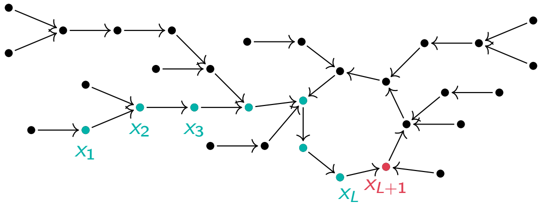


$M = \beta^{L+1}$



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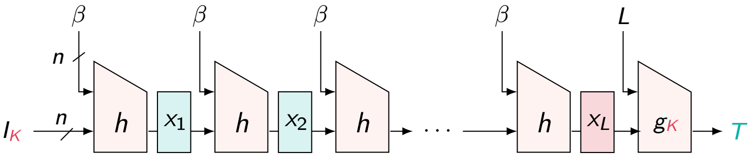

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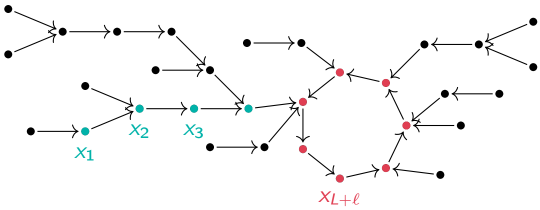

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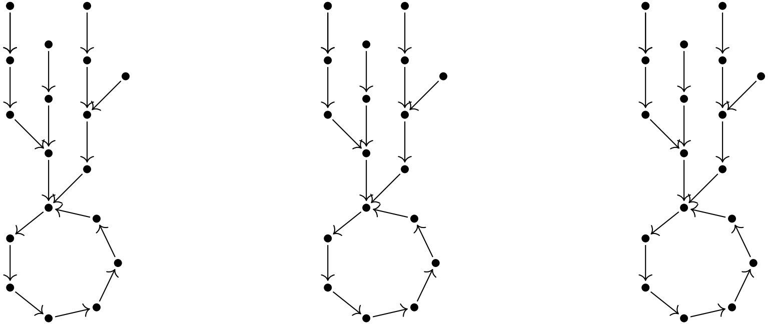

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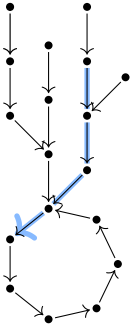

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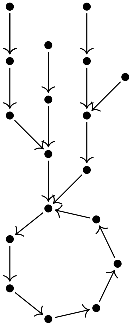

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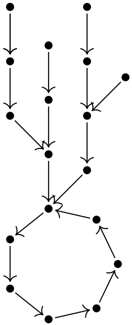

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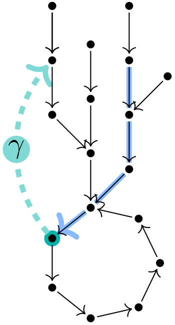

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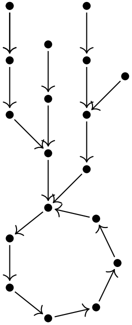

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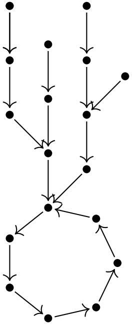

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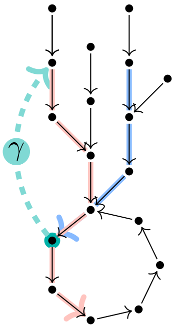

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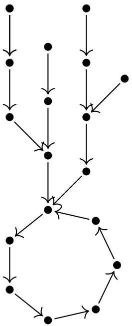

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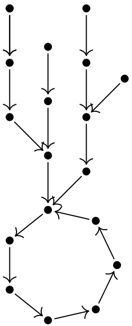

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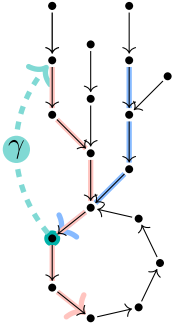

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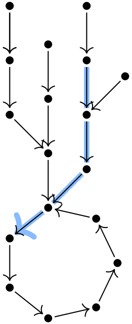

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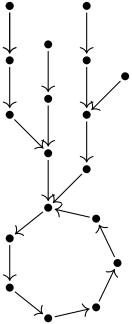

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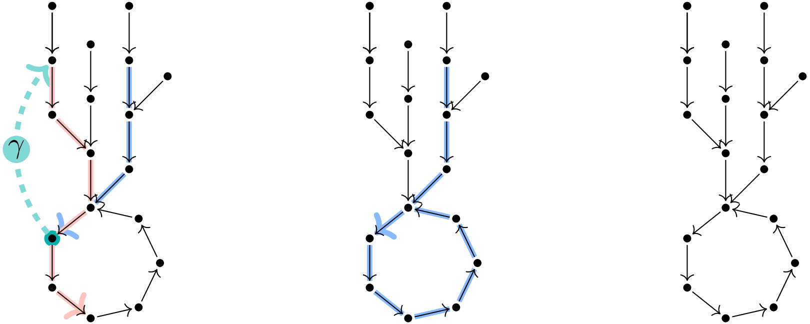

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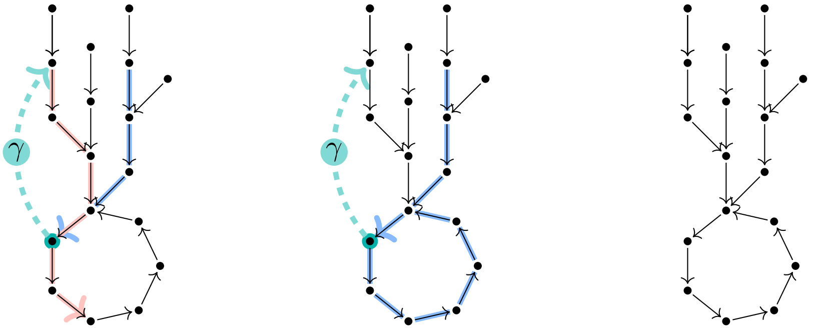

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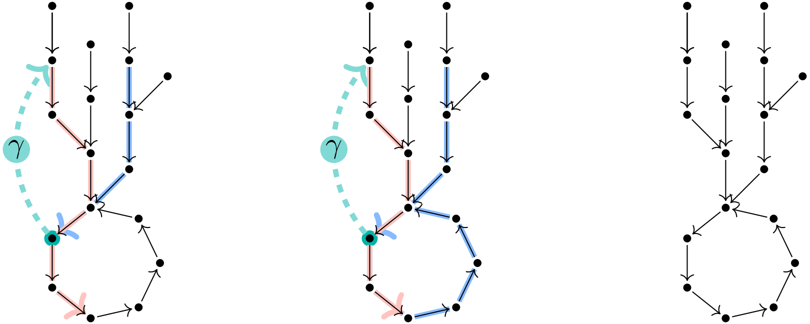

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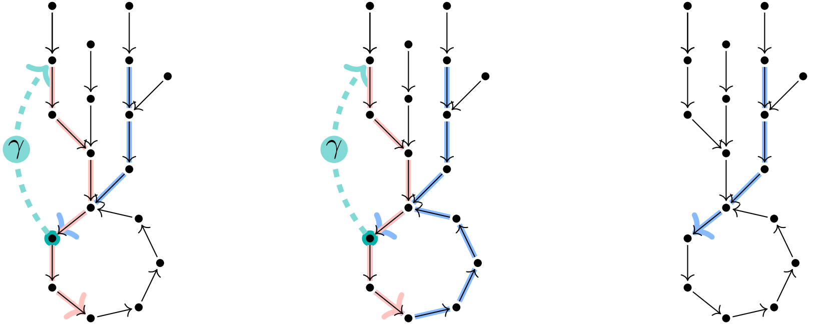

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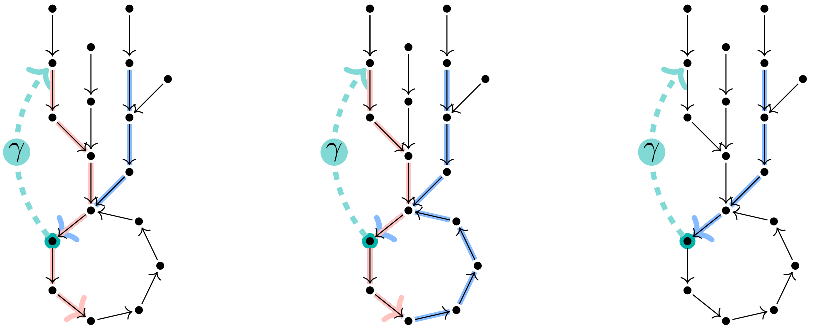

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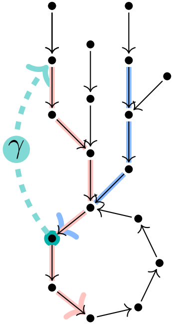

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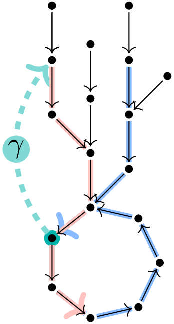

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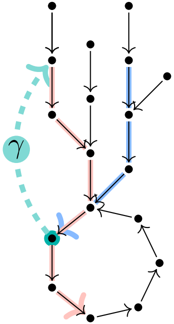

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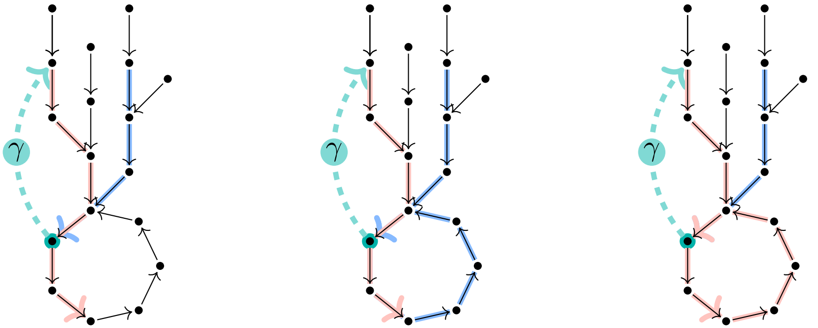

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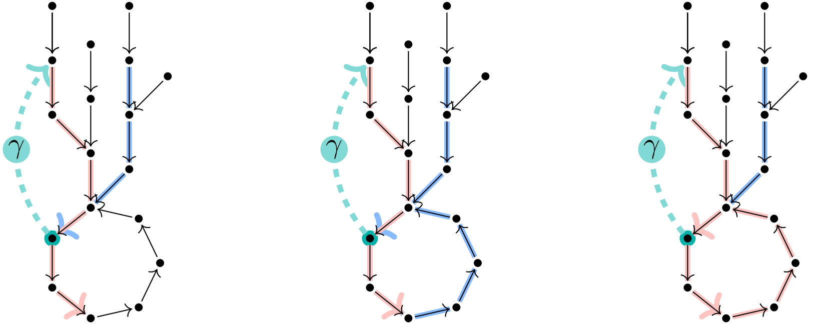

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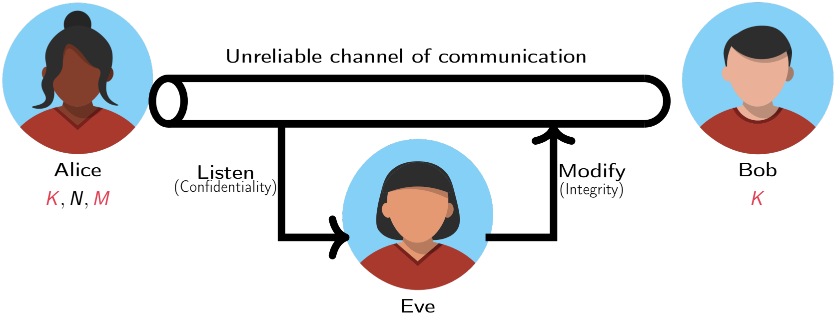
Still no state recovery. Idea 3: use the root of the main tree α.

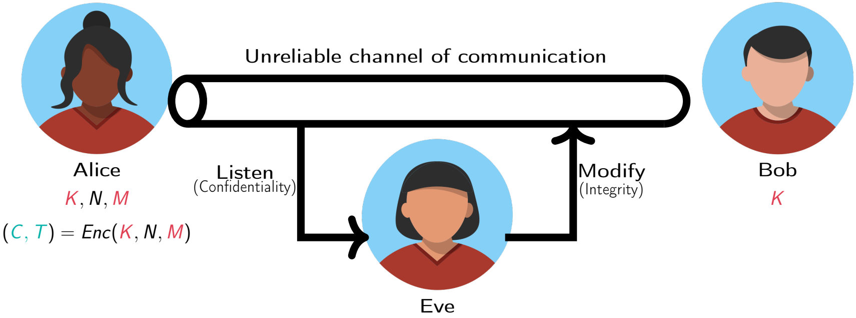
Outline

Authenticated Encryption



An authenticated encryption scheme ensures both the confidentiality and integrity of communications.

Authenticated Encryption



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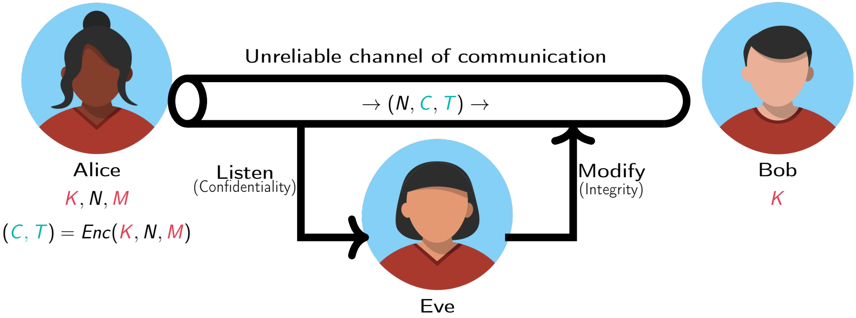

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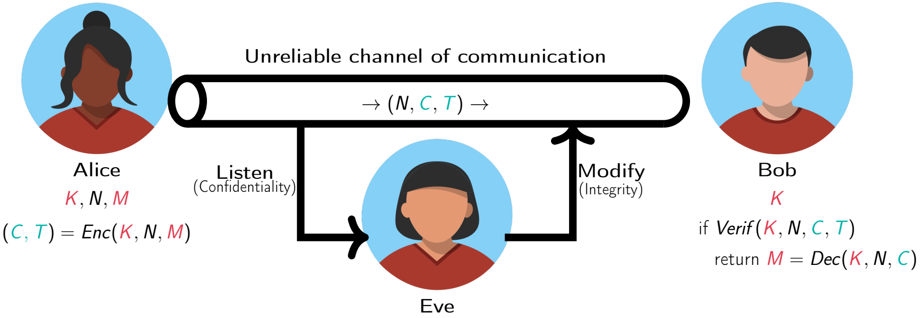

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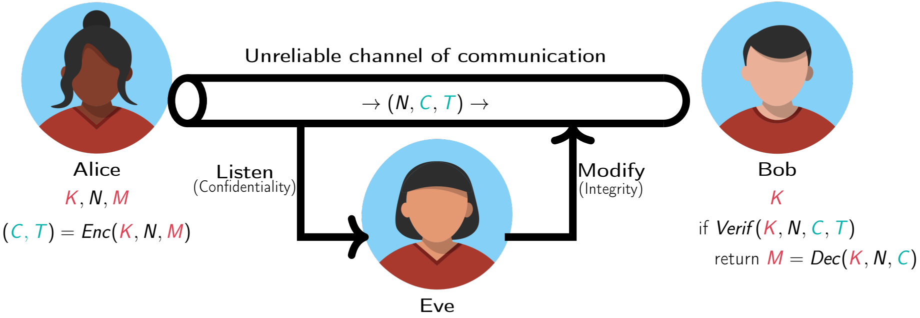

Forgery attack: find a decryption query (N, C, T) s.t. the tag verification succeeds.

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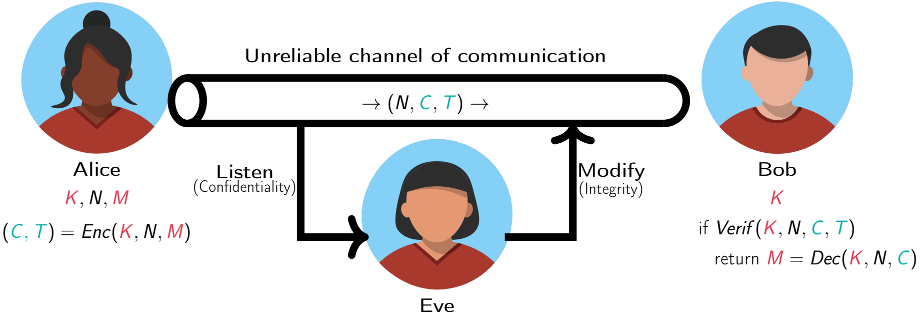


Forgery attack: find a decryption query (N, C, T) s.t. the tag verification succeeds.

- Assuming a nonce-respecting adversary
- and no release of unverified plaintext.

Duplex-based AE modes

Authenticated Encryption

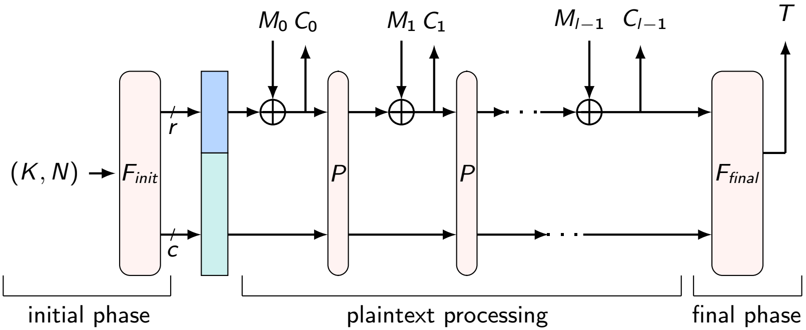
- (Historically) block-cipher based: (tweakable) block cipher + mode
- (More recently) permutation-based: public permutation + keyed mode

Permutation-based modes of operation [BDPVA11]

- Many candidates at the NIST lightweight competition (2018-2023), including the winner ASCON.
- Modes are proven secure when instantiated with a random permutation.
- It is difficult to assess in practice $\rightarrow$ cryptanalysis.

Duplex-based AE modes [BDPVA11,DMV17]

Encryption

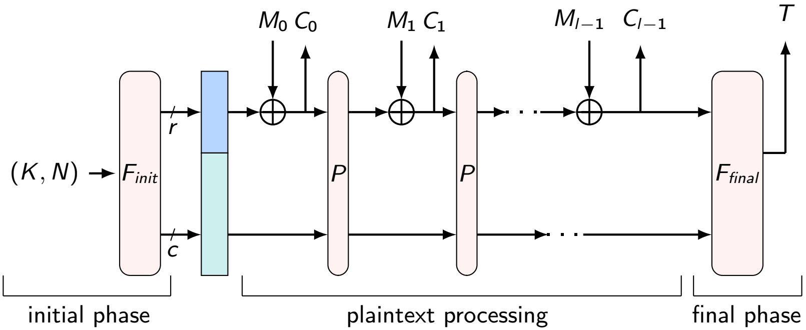


- Permutation P operates on a state of length $b = r + c$ bits, r is the rate, c the capacity.
- First r bits: the outer state
- Next c bits: the inner state

Ex: Cyclist (Xoodyak)
$r = 192$, $c = 192$

Duplex-based AE modes [BDPVA11,DMV17]

Encryption



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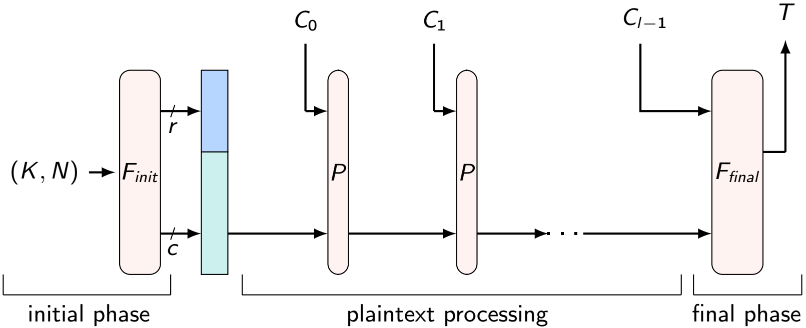

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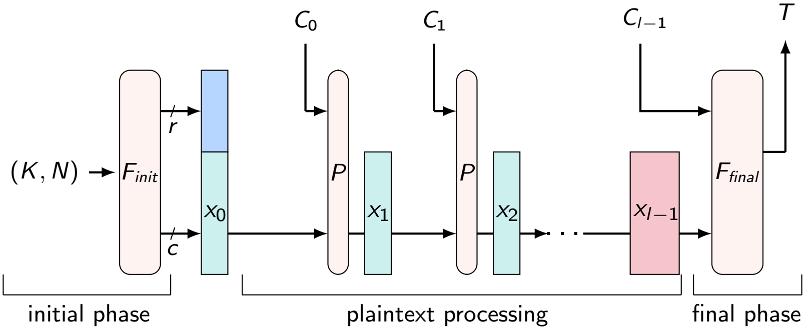
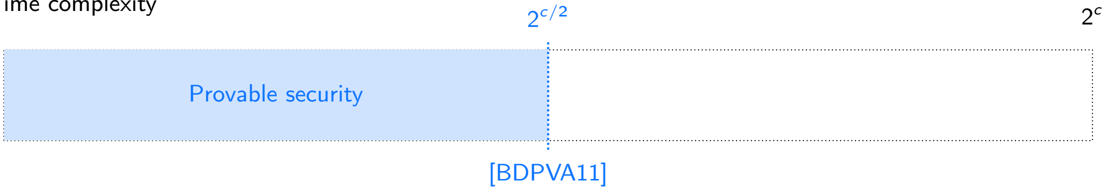


The knowledge of x_{l-1} allows to build a forgery.

Security of duplex-based modes

Assuming a sufficiently large key/tag/state length:

Time complexity



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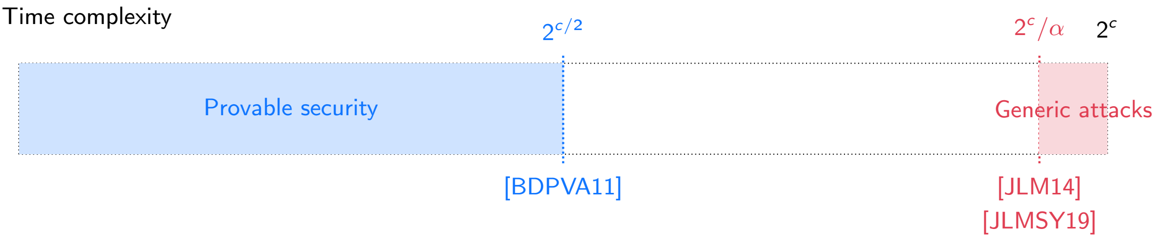

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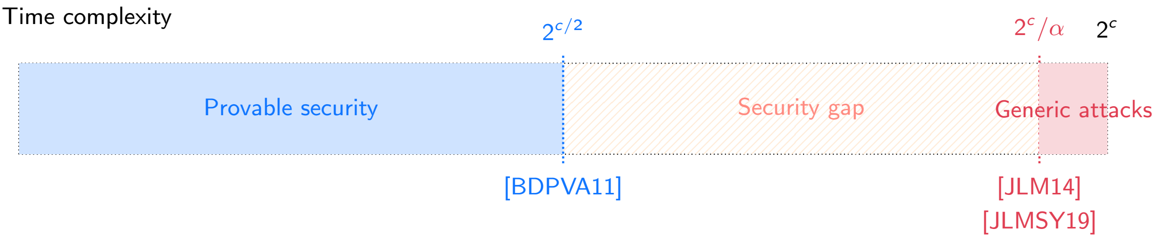

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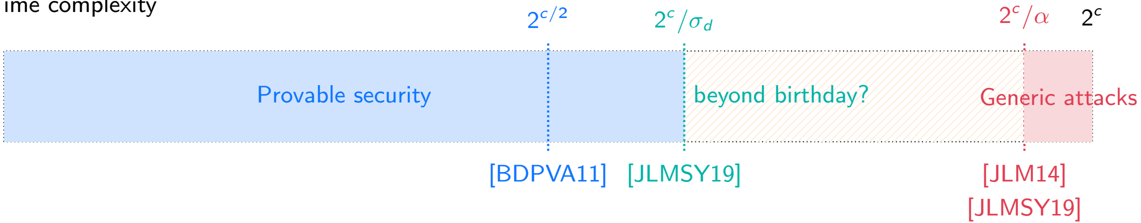

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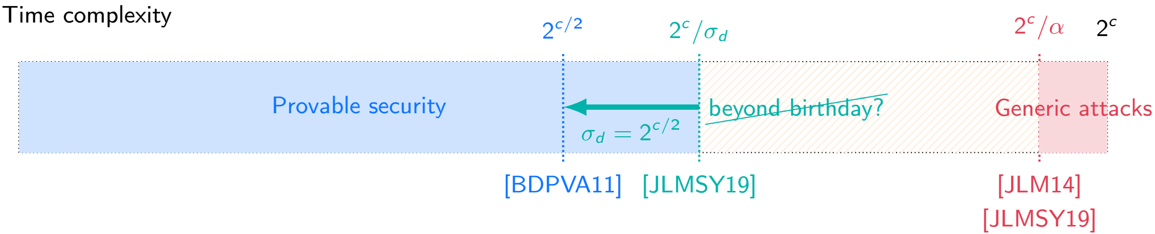

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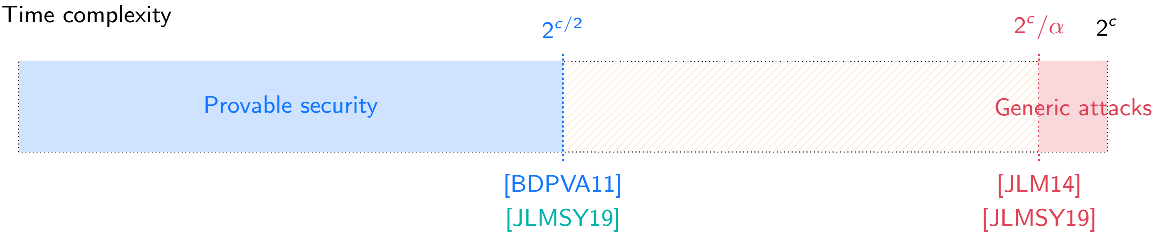

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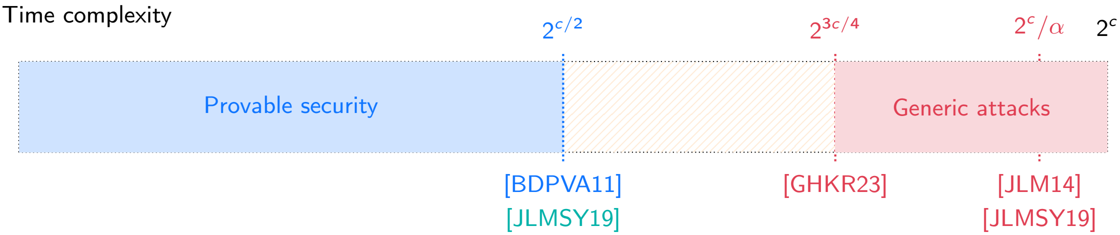

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Generic Attack on Duplex-Based AEAD Modes Using Random Function Statistics. Gilbert, Heim Boissier, Khati, Rotella. **EUROCRYPT 2023**

Disclaimer: this is simplified

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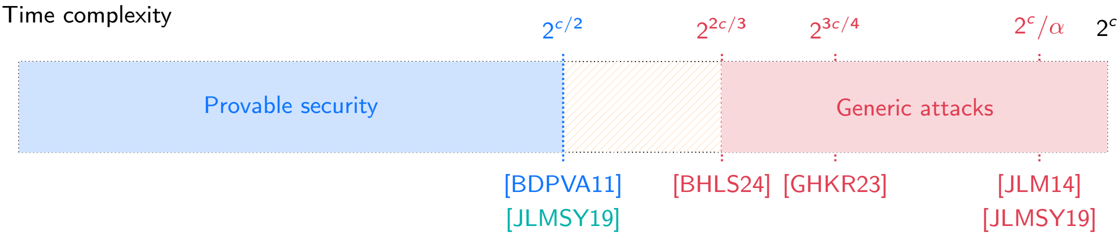

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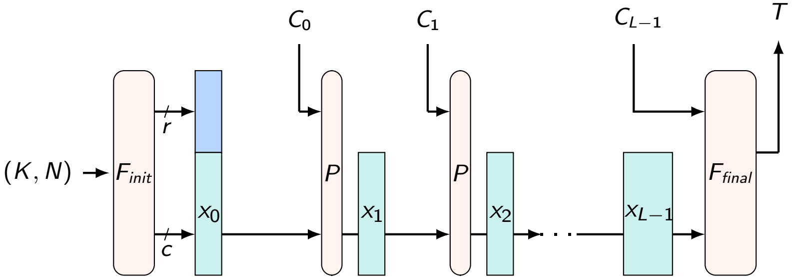
Improving Generic Attacks Using Exceptional Functions. Bonnetain, Heim Boissier, Leurent, Schrottenloher. **CRYPTO 2024**

Main observation (1/2)

Verification $(C = C_0 \,||\, \cdots || C_{L-1}, T)$



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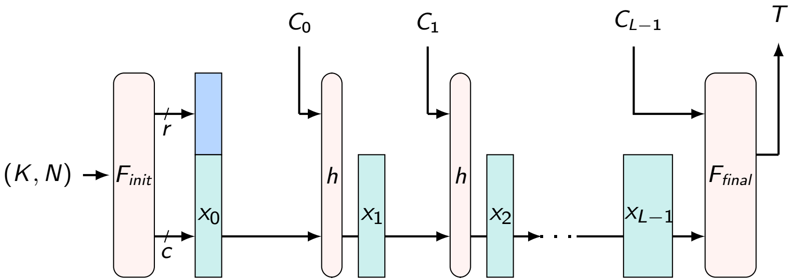

We define a compression function h induced by P:

$$\begin{aligned} h : \mathbb{F}_2^b &\longrightarrow \mathbb{F}_2^c \\ x &\longmapsto \lfloor P(x) \rfloor_c \,. \end{aligned}$$

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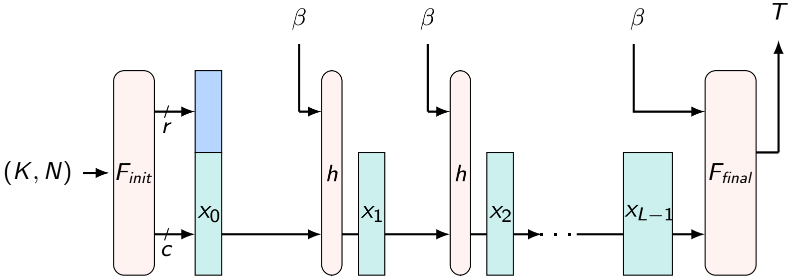
Verification (β^L, T)



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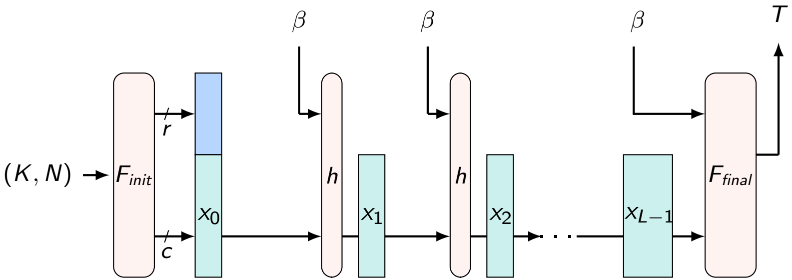
Main observation (1/2)

Verification (β^L, T)

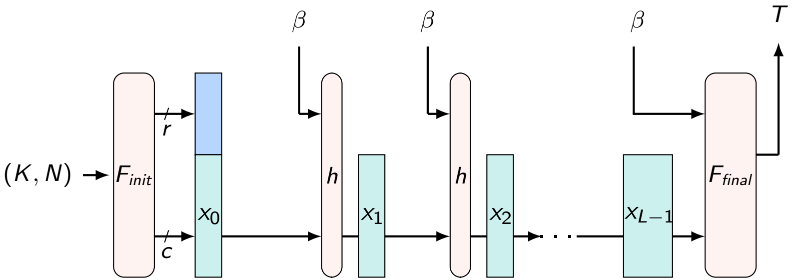


The tag verification iterates the function

$$\begin{aligned} h_\beta : \mathbb{F}_2^c &\longrightarrow \mathbb{F}_2^c \\ x &\longmapsto h(\beta, x)\,. \end{aligned}$$

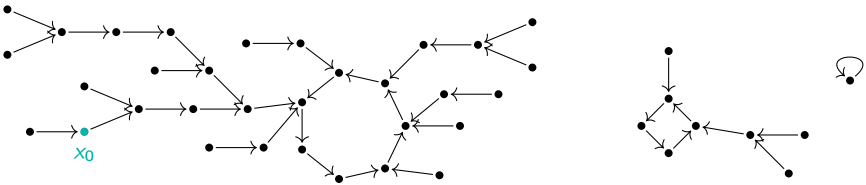
Main observation (1/2)

Verification (β^L, T)

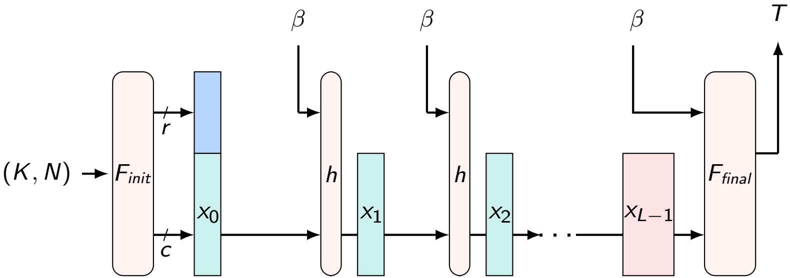


- For a random β, we expect h_β to behave as a random function drawn in $\mathfrak{F}_{2^c}$.
- For each nonce, we expect x_0 to behave as a random point drawn in the graph of h_β.

Main observation (2/2)



Graph of h_β



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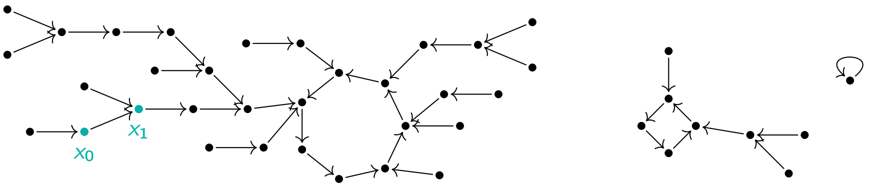

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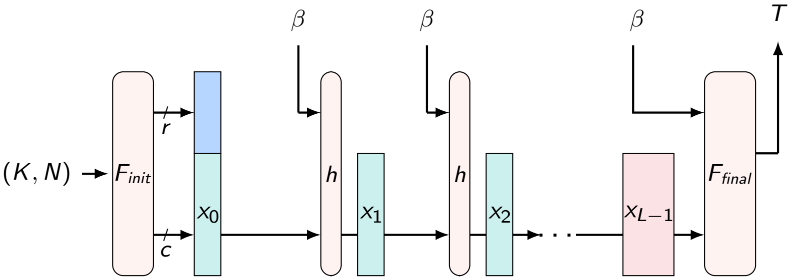
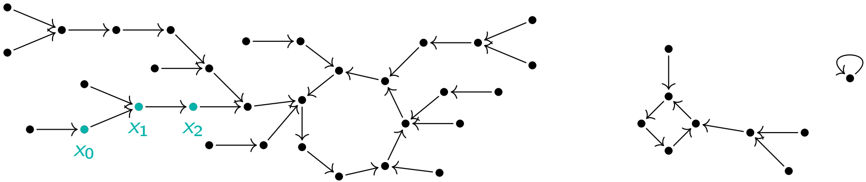


Main observation (2/2)



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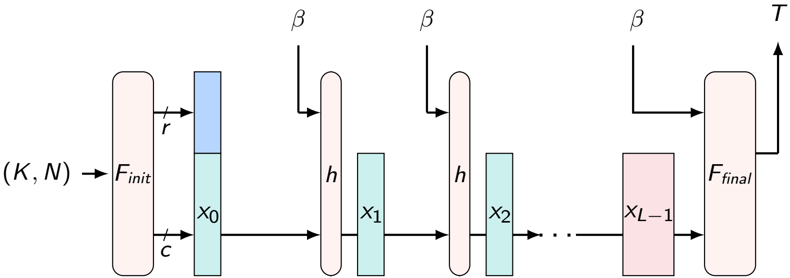

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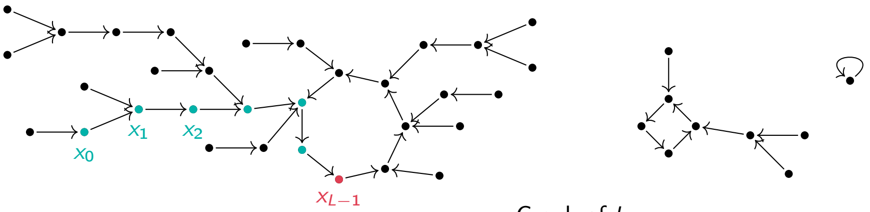
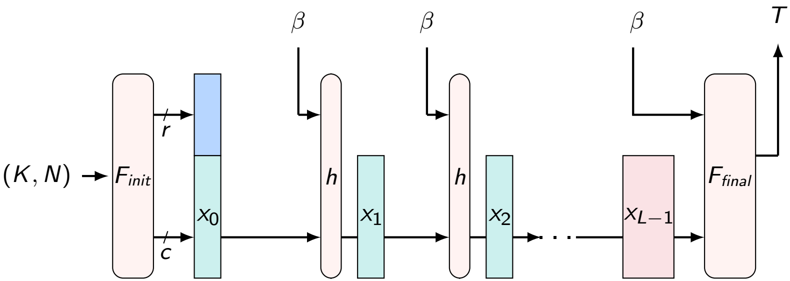
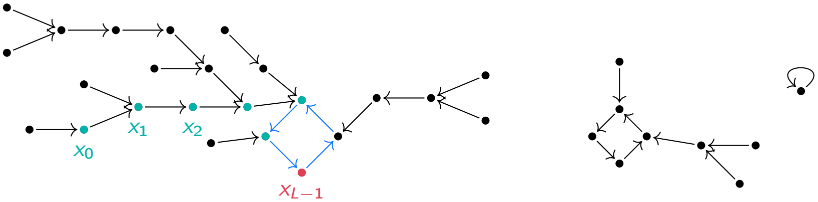


Graph of h_β



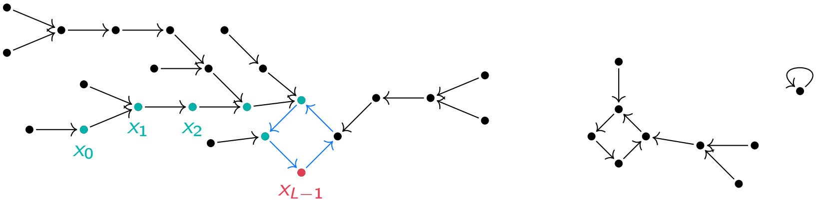
Exceptional functions



Graph of an exceptional h_β

If one finds β s.t. h_β has a reasonably large component (say $\geq 0.65 \cdot 2^c$) with an exceptionnally small cycle (say $\leq 2^{\frac{c}{4}}$)...

Exceptional functions



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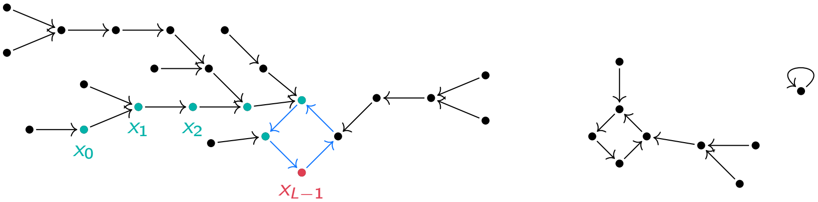

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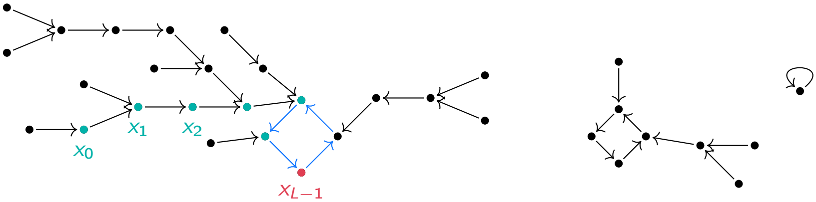
- If so, if L is 'large enough' ($L = cst \cdot 2^{\frac{c}{2}}$), x_{L-1} is in the small cycle with good probability.

Exceptional functions



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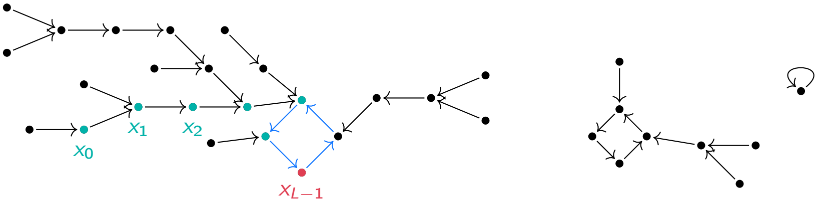
Exceptional functions



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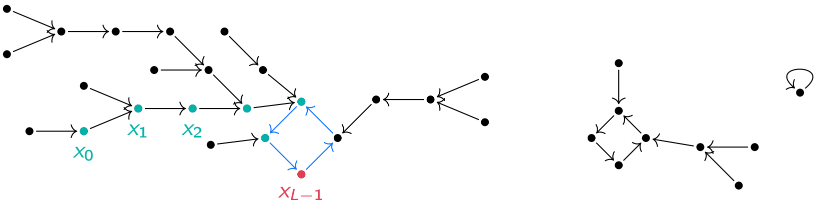
- If so, if L is 'large enough' ($L = cst \cdot 2^{\frac{c}{2}}$), x_{L-1} is in the small cycle with good probability.
- If so, there are at most $2^{\frac{c}{4}}$ possible values for x_{L-1}; i.e., at most $2^{\frac{c}{4}}$ possible tags.

Resulting forgery attack: (1) precompute an exceptional h_β and (2) try the $\leq 2^{\frac{c}{4}}$ possible values for T.

A new statistic



Graph of an exceptional h_β

[DeLaurentis87]: Probability that a h_β has a component s.t.

- (exceptionally small cycle) $\ell \leq 2^\mu$ (e.g. $\ell \leq 2^{-c/4}$);
- (reasonably large size) of size $\geq 2^c \cdot s$ (e.g. size $\geq 0.65 \cdot 2^c$):

$$p_{s,\mu} \approx \sqrt{2(1-s)/\pi s} \cdot 2^{\mu - \frac{c}{2}} \qquad \text{(e.g. } 0.6 \cdot 2^{-\frac{c}{4}}\text{).}$$

Forgery attack [GKHR23]

1 **Precomputation phase**: Find β s.t. h_β has a large component ($\geq 0.65 \cdot 2^c$) with an exceptionally small cycle ($\leq 2^\mu$) and recover this cycle.

- For random β's,
  - Recover the cycle length using Brent's algorithm.

\# candidates for $\beta \approx 1/p_{s,\mu} \approx 2^{\frac{c}{2}-\mu}$
complexity $\approx 2^{\frac{c}{2}}$ applications of h

Total complexity: $\approx 2^{c-\mu}$ applications of h.

2 **Online phase**: Submit $(N, C = \beta^L, T)$ queries where $T = F_{final}(\beta || x)$, x in the cycle.

- Set $L = 3 \cdot 2^{\frac{c}{2}}$ so that x_{L-1} in the cycle with good probability
- At most 2^μ possible values for T.

Total complexity: $\approx 2^{\frac{c}{2}+\mu}$ applications of h.

Balanced complexity: $2^{\frac{3c}{4}}$

Summary of our results

Beyond an asymptotic result

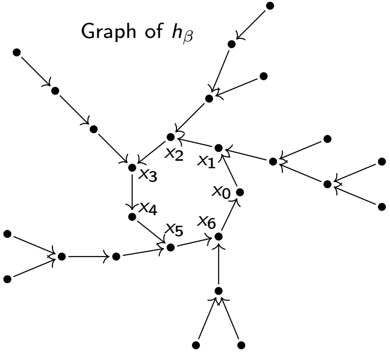
- Total time complexity: $\leq 21 \cdot 2^{\frac{3c}{4}}$.
- Probability of success: $\geq 95\%$.
- NB: almost always a key recovery (since forgery $\rightarrow$ state recovery $\rightarrow$ key recovery).

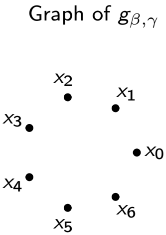
Applications

- Modes of `Norx` v2, Ketje, KNOT and Keyak;
- Attack of complexity 2^{148} on Xoodyak
  - Breaks a 184-bit security claim (corrected since).

A new improvement [BHLS24]

Define a nested function $g_{\beta,\gamma}$ from the cycle $\mathscr{C}$ of h_β to itself.





$g_{\beta,\gamma} = h_\beta^L \circ h_\gamma : x \in \mathscr{C} \longmapsto x' \in \mathscr{C}$ with good probability.

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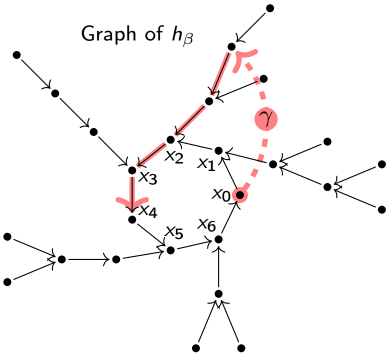

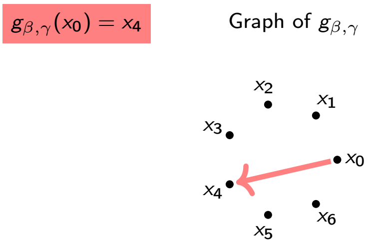

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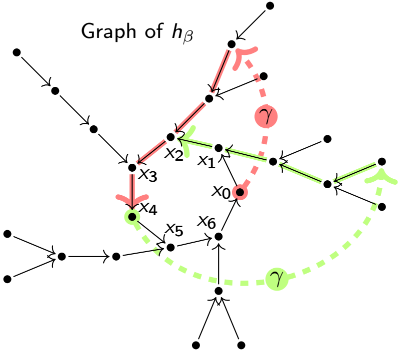
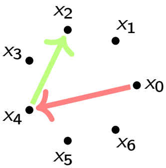


$g_{\beta,\gamma}(x_0) = x_4$

$g_{\beta,\gamma}(x_4) = x_2$



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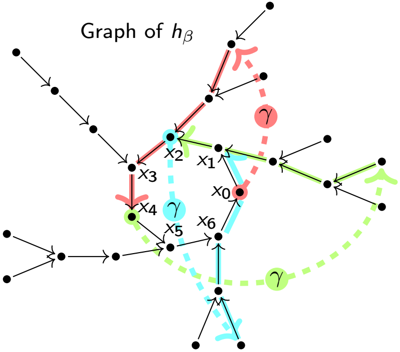
Graph of h_β



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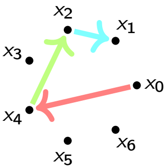

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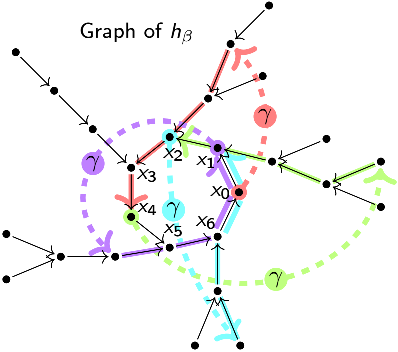

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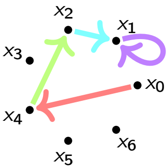

$g_{\beta,\gamma} = h_\beta^L \circ h_\gamma : x \in \mathscr{C} \longmapsto x' \in \mathscr{C}$ with good probability.

Nesting exceptional functions [BHLS24]

1. Find β s.t. h_β is exceptional.
   - Let $2^\mu \ll 2^{c/2}$ be the cycle length of h_β.
2. Find γ s.t. $g_{\beta,\gamma}$ is exceptional.
   - For a *random* γ, $g_{\beta,\gamma}$ has cycle length $2^{\mu/2} \ll 2^{c/4}$.
   - Let $2^\nu (\ll 2^{c/4})$ be the cycle length of the exceptional $g_{\beta,\gamma}$.
3. One must only try 2^ν tags, but the ciphertexts are a lot longer.

For $\mu = 2c/7$ and $\nu = c/14$, the balanced total complexity is $2^{5c/7} < 2^{3c/4}$.

Our best attack against duplex-based modes has complexity $2^{2c/3}$.

- It uses precomputations in the graph of h_β.

Other contributions [BHLS24]

- Generic attacks against hash combiners using (nested) exceptional functions.
- Historically, those attacks use a bunch of cryptanalytic tools.
  - Joux's multi-collisions, Diamond structure, Expandable messages,...
- Using (classical) exceptional functions, we improve the best existing attacks against
  - XOR Combiner. $M \mapsto H_1(M) \oplus H_2(M)$ (preimage);
  - Zipper Hash. $M \mapsto H_2(H_1(\mathrm{IV}, M), \overleftarrow{M})$ (second preimage);
  - Hash-Twice. $M \mapsto H_2(H_1(\mathrm{IV}, M), M)$ (second preimage, second preimage quantum).

Outline

Key take-aways

Functional graphs have many applications in generic cryptanalysis.

Our contribution [GKHR23,BHLS24]

- Showing the applicability of functional graph techniques to AE modes.
- First use of exceptional behaviour of random functions.
- Bridging the gap between provable security and practical attacks.
  - A variant of our attack w/ computational complexity $O(2^c)$ is 'tight'. [Lef24]
- Beyond asymptotic results: break of a security assumption of Xoodyak.
- Improving a long series of attacks on hash combiners.

Perspectives and fun follow-up questions

Fully specified primitives

- Finding exceptional functions on real-life permutations using their specification.
- Building a backdoor permutation that 'looks' secure, but with a known exceptional function.

Overall goal: Bridging the gap between provable security and cryptanalysis.

- What about the quantum setting?

Removing residual heuristics

- Heuristic assumptions on the distribution of $t(x_0)$ for x_0 in an exceptional component.
- Experimentally verified.

Thank you for your attention!